University of California, San Diego
ECE 45
Midterm Exam

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Print your name : ______________________________
Student ID Number : ______________________________

- No Books, No Notes, No calculators allowed

Question	Score
1	
2	
3	
4	

Some Useful Formulas:

$$Ae^{j\theta} = A\cos(\theta) + jA\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad , \quad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \quad , \quad C_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$, $y(t)$ } Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t)$, $\alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a^*_{-k}$ $\Re e\{a_k\} = \Re e\{a_{-k}\}$ $\Im m\{a_k\} = -\Im m\{a_{-k}\}$ $\|a_k\| = \|a_{-k}\|$ $\sphericalangle a_k = -\sphericalangle a_{-k}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

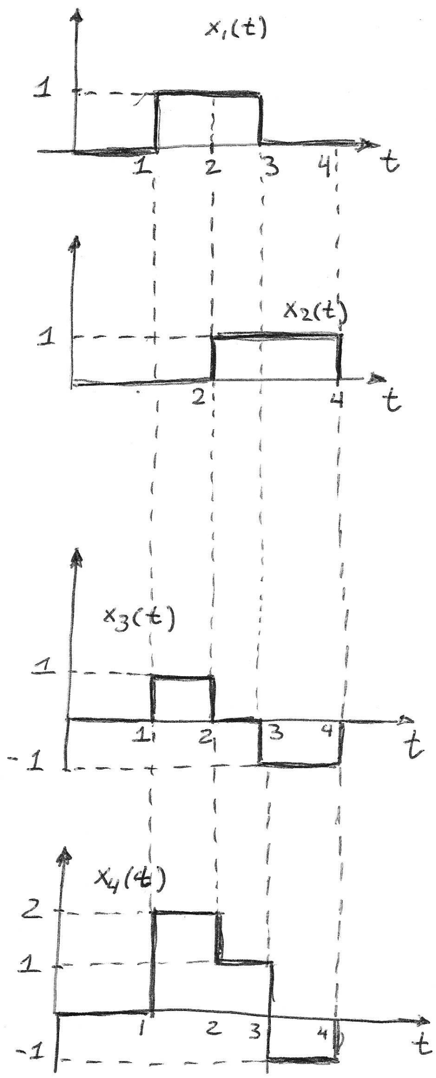
Problem 1) [15 pts]

Assume that you measured the response y1(t) of an LTI system to the signal x1(t) indicated below.

It is now late into the night, you are far from the lab, and just realized that to complete your project you also need to know the responses to signals x2(t), x3(t), x4(t).

Your collaborators panic, but you have taken ECE45 and claim that without making any additional measurements you can write these responses in terms of y(t).

Prove that this is the case by writing the response to x2(t), x3(t), x4(t) in terms of the signal y1(t).



$$x_1(t) = \begin{cases} 1 & \text{for } t \in [1,3] \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1 & \text{for } t \in [2,4] \\ 0 & \text{otherwise} \end{cases}$$

$$x_3(t) = \begin{cases} 1 & \text{for } t \in [1,2] \\ -1 & \text{for } t \in [3,4] \\ 0 & \text{otherwise} \end{cases}$$

$$x_4(t) = \begin{cases} 2 & \text{for } t \in [1,2] \\ 1 & \text{for } t \in [2,3] \\ -1 & \text{for } t \in [3,4] \\ 0 & \text{otherwise} \end{cases}$$

Problem 2) [30 pts]
Draw the Bode plots (amplitude and phase) for the following frequency response

$$H(w) = 2500\frac{(j\omega)^2 + j\omega}{(j\omega + 10)^2(j\omega + 500)}$$

Problem 3) [55 pts]

a) [10 pts] Compute the average power of the periodic signals x1(t) and x2(t) depicted below.

b) [20 pts] Determine the Fourier series coefficients for x1(t).
Hint: Recall the integration by parts formula:

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

Rather than using the formula twice, you may use superposition and write the signal as the sum of two signals

c) [15 pts] Without performing the computation again, but using the properties of the Fourier series on page 2 of the exam, also determine the coefficients for x2(t).

d) [10pts] Write the response to signals x1(t) and x2(t) of an LTI system defined by

$$H(w) = 20\frac{(j\omega)}{(1+j\omega)}$$