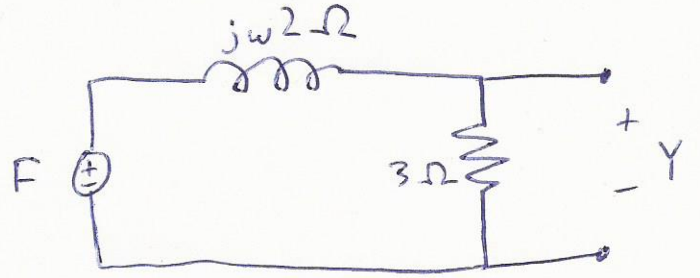
Midterm 1 Solutions ECE 45

(1.) Phasorize circuit:



use voltage division:

$$Y = \frac{3}{3+j\omega 2} F \longrightarrow \frac{Y}{F} = H(\omega) = \frac{3}{3+j\omega 2}$$

multi-frequency input: (0, 2, 4)

$H(0) = 1$

$H(2) = \dfrac{3}{3+4j} = \dfrac{3}{5} \angle -53.13^\circ$

$H(4) = \dfrac{3}{3+8j} = 0.35 \angle -69.44^\circ$

OK to leave as fractions or $\tan(b/a)$

$$\Rightarrow \quad y(t) = |H(0)| \cdot 5 + |H(2)| \sin(2t + \angle H(2)) + |H(4)| \cos(4t + \angle H(4))$$

$$y(t) = 5 + 0.6 \sin(2t - 53.13^\circ) + 0.35 \cos(4t - 69.44^\circ)$$

OK as fractions or $\tan(b/a)$

(2) $H(\omega) = 10^6 \dfrac{(j\omega)^2 + j\omega}{(j\omega+10)(j\omega+10)(j\omega+100)} = 100 \dfrac{j\omega(j\omega+1)}{\left(\frac{j\omega}{10}+1\right)^2 \left(\frac{j\omega}{100}+1\right)}$

Components:

(1) Constant of 100

(2) zero $[j\omega]$

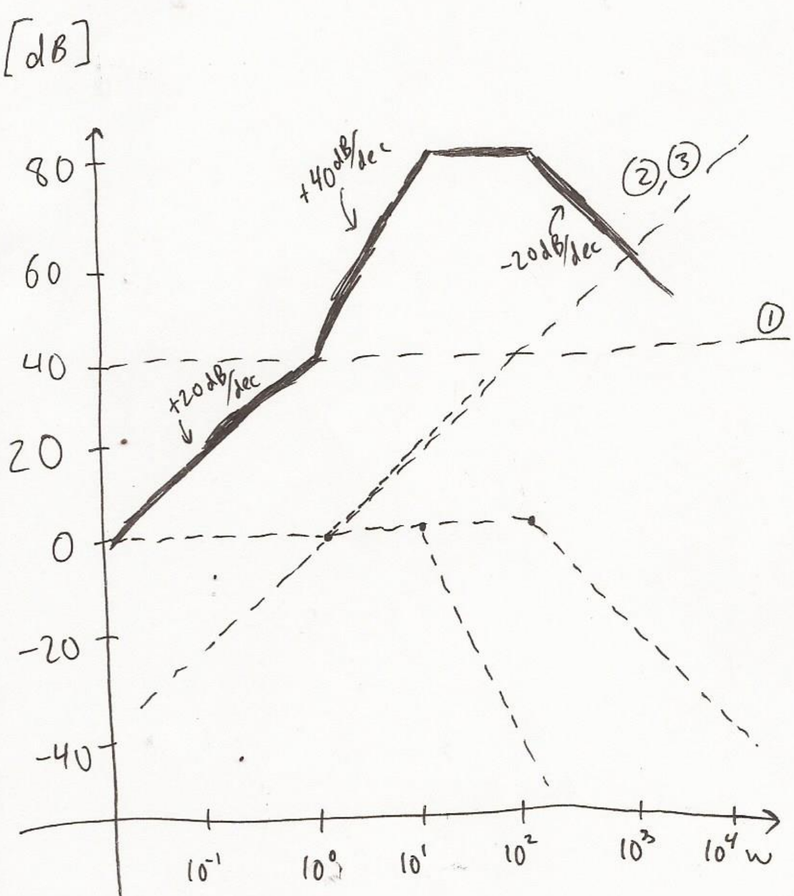
(3) zero $[j\omega+1]$

(4) pole $\left[\frac{j\omega}{100}+1\right]^{-1}$

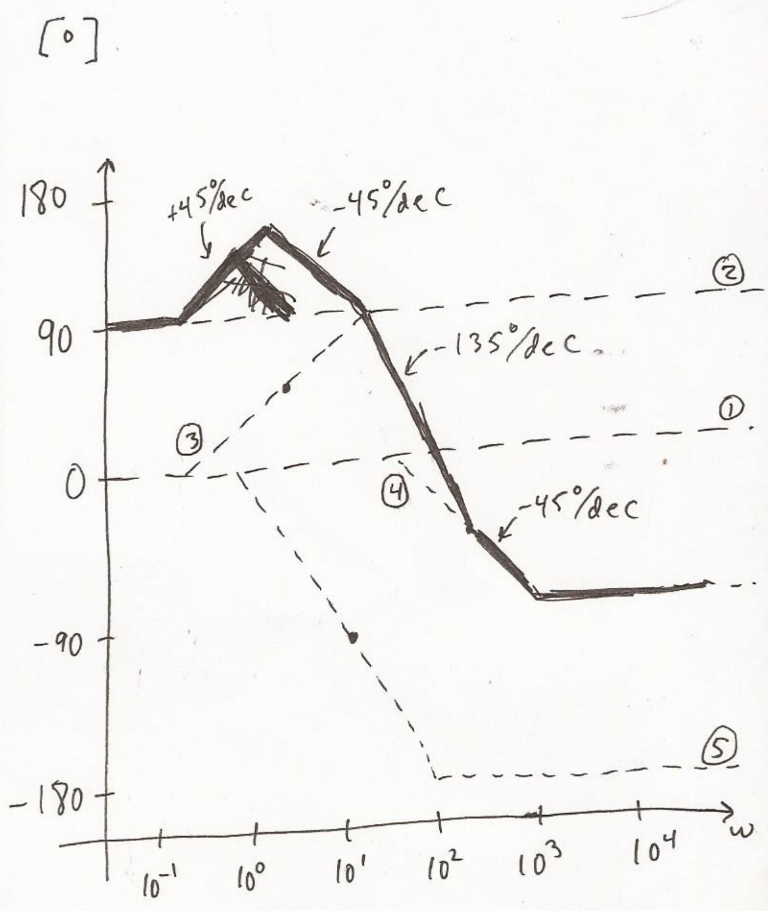
(5) double-pole $\left[\frac{j\omega}{10}+1\right]^{-2}$

Bode Plot:

Magnitude:



Phase:



3. (a) $T_0 = 4$, $\omega_0 = \frac{\pi}{2}$

$$X_{1,n} = \frac{1}{4}\int_0^4 x_1(t)\, e^{-jn(\frac{\pi}{2})t}\, dt$$

$$X_{1,n} = \frac{1}{4}\left[\int_0^1 e^{-jn\frac{\pi}{2}t}\, dt - \int_1^2 e^{-jn\frac{\pi}{2}t}\, dt + \cancelto{0}{\int_2^4 0\, dt}\right]$$

$$X_{1,n} = \frac{1}{4}\left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\Bigg|_0^1 - \frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\Bigg|_1^2\right] = \frac{e^{-j\frac{n\pi}{2}} - 1 - e^{-jn\pi} + e^{-j\frac{n\pi}{2}}}{-jn2\pi} = \frac{1 + e^{-jn\pi} - 2e^{-jn\frac{\pi}{2}}}{2n\pi j}$$

(further simplification optional)

(b) This function $x_2(t)$ is related to $x_1(t)$:

$$\frac{d\, x_2(t+t_0)}{dt} = x_1(t), \quad t_0 = -2$$

So, use properties: $X_{1,n} = jn\frac{\pi}{2} X_{2,n}^{\text{no shift}} \rightarrow X_{2,n}^{\text{no shift}} = \frac{X_{1,n}}{jn\frac{\pi}{2}}$

$$x_2(t) = x_2^{\text{no shift}}(t - t_0), \quad t_0 = -2$$

$$\Rightarrow X_{2,n} = X_{2,n}^{\text{no shift}}\, e^{-jn\frac{\pi}{2}(-2)} \Rightarrow X_{2,n} = \frac{X_{1,n}}{jn\frac{\pi}{2}}\, e^{jn\pi}$$ (plugging in $X_{1,n}$ optional)

(c) This function $x_3(t)$ is the sum $x_1(t) + x_2(t) = x_3(t)$

So, use properties:

$$X_{3,n} = X_{1,n} + X_{2,n}$$ (plugging in $X_{1,n}$ and $X_{2,n}$ optional)

4.

$$a_k = \begin{cases} 2, & k=0 \\ j\left(\frac{1}{2}\right)^{|k|}, & k \neq 0 \end{cases}$$

(problem 3.26 in book)

(a) If $x(t)$ is real, then $x(t) = x^*(t)$

This implies that for $x(t)$ real, $a_k = a^*_{-k}$

Since this is not true, $x(t)$ is not real

(b) If $x(t)$ is even, then $x(t) = x(-t)$, and $a_k = a_{-k}$

Since this is true, $x(t)$ is even

(c)

$$g(t) = \frac{dx(t)}{dt} \xleftrightarrow{F.S.} b_k = jk\frac{2\pi}{T_0}a_k$$

$\omega = 0,\ T \to \infty$

$$\Rightarrow b_k = \begin{cases} 0, & k=0 \\ -k\frac{2\pi}{T_0}\left(\frac{1}{2}\right)^{|k|}, & k \neq 0 \end{cases}$$

b_k is not even $\Rightarrow \frac{dx(t)}{dt}$ is not even