University of California, San Diego

ECE 45

Midterm - I

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Print your name: Solutions

Student ID number: ______

Note: No books, no notes, no calculators allowed.

Question	Score
1	
2	
3	
Total	

1. Consider the frequency response:

$$H(\omega) = \frac{2 \cdot 10^{-3} j\omega(100 + j\omega)(2 \cdot 10^4 + j2\omega)}{(40 + j4\omega)(1 + \frac{j\omega}{10^3})^2}$$

(a) Represent $H(\omega)$ in the standard form i.e.

$$\frac{c(j\omega)(1 + j\omega/a_1)...(1 + j\omega/a_k)}{(1 + j\omega/b_1)...(1 + j\omega/b_k)}$$

(5 points)

(b) Draw the Bode Plot of $H(\omega)$. **(15 points)**

a) $$H(\omega) = \frac{2\times10^{-3} \times 100 \times 2\times10^4 \, (j\omega)\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{10^4}\right)}{40\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^3}\right)^2}$$

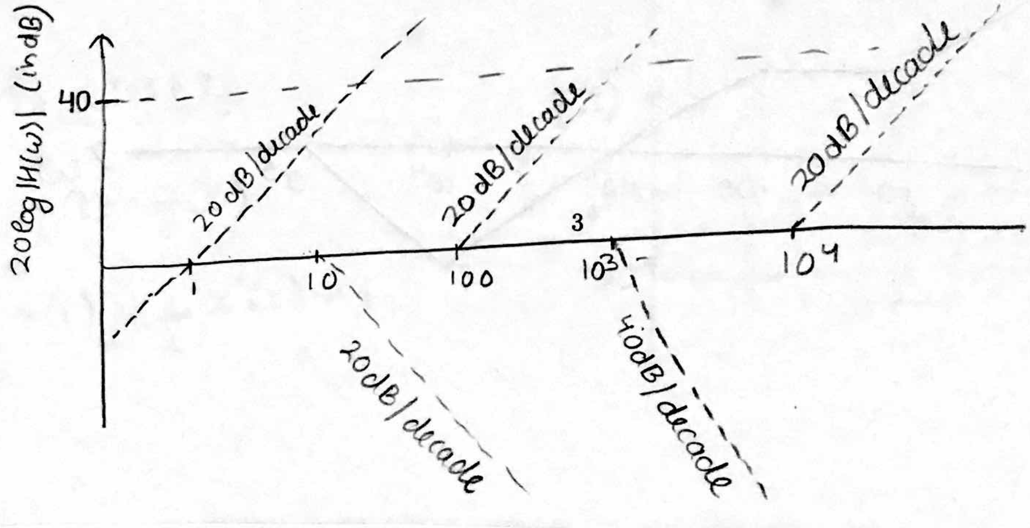
$$= \frac{100\,(j\omega)\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{10^4}\right)}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^3}\right)^2}$$

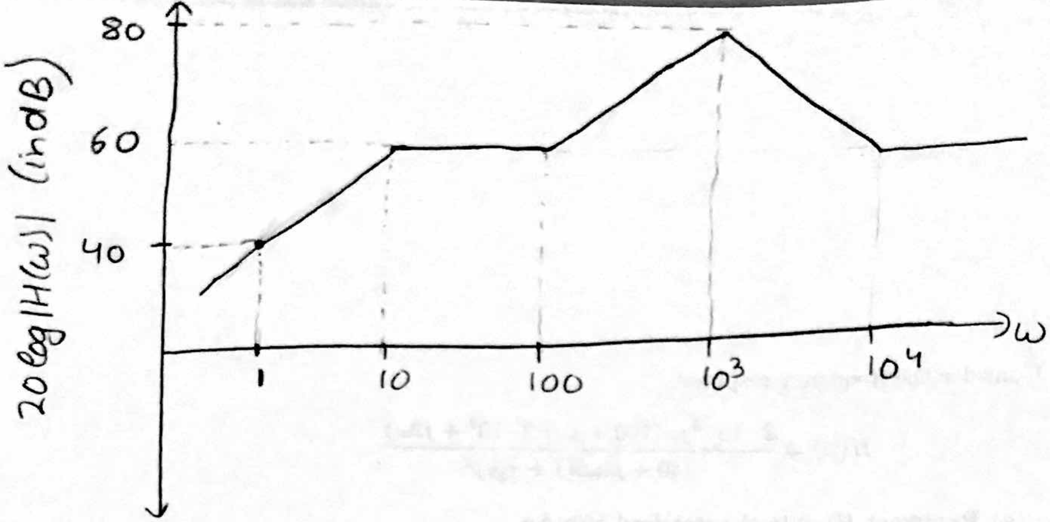
b)

$$20\log|H(\omega)| = 20\log 100 + 20\log|\omega| + 20\log\sqrt{1+\frac{\omega^2}{(100)^2}}$$

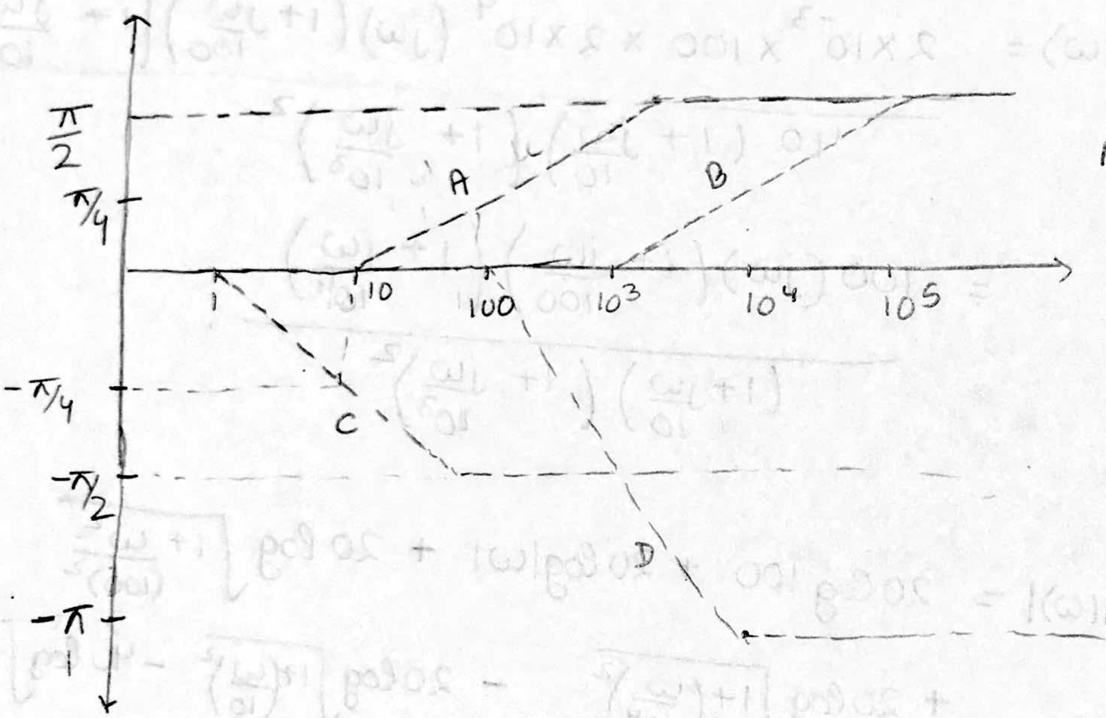
$$+ 20\log\sqrt{1+\left(\frac{\omega}{10^4}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2} - 40\log\sqrt{1+\left(\frac{\omega}{10^3}\right)^2}$$

critical ω = 1, 10, 100, 10^3, 10^3, 10^4





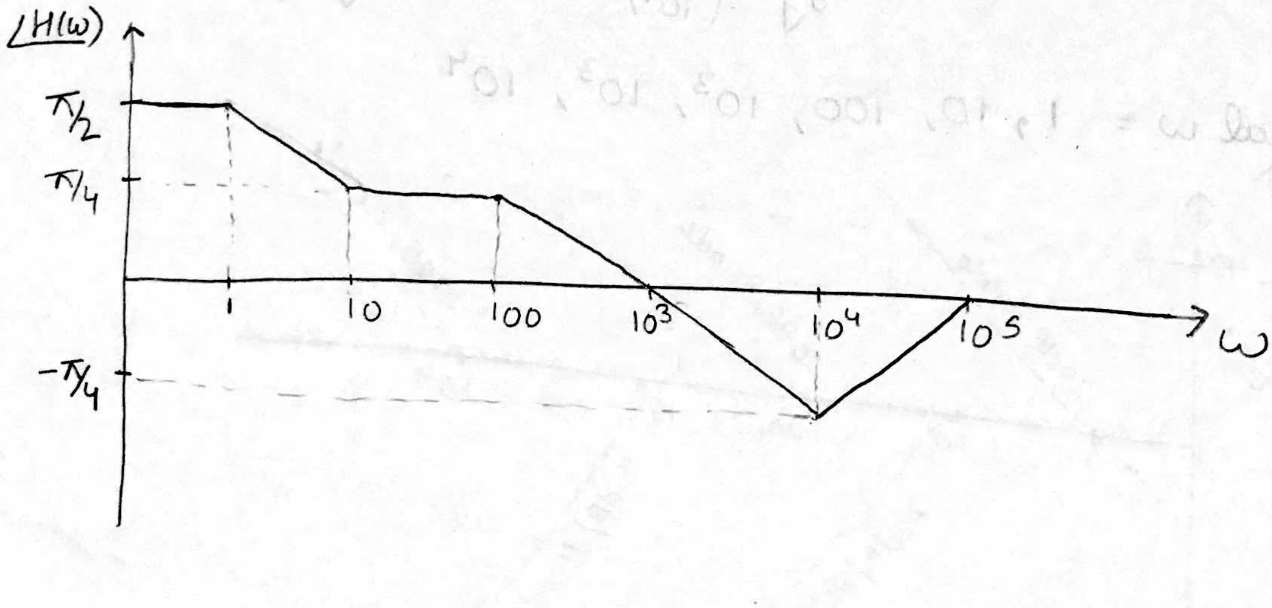
$$\angle H(\omega) = \frac{\pi}{2} + \tan^{-1}\left(\frac{\omega}{100}\right) + \tan^{-1}\left(\frac{\omega}{10^4}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - 2\tan^{-1}\left(\frac{\omega}{10^3}\right)$$



Slope

A, B, C: $\frac{\pi}{4}$ rad/decade

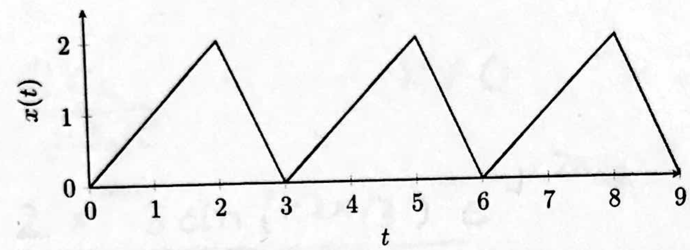
D: $\frac{\pi}{2}$ rad/decade

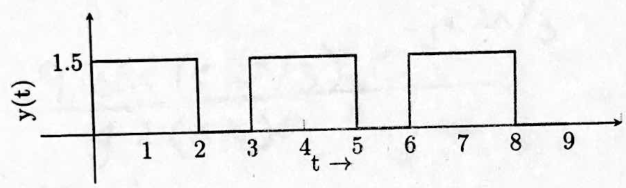


(Note: Similar to HW 2 question 9.)

2. Find the Fourier Series coefficients of $x(t)$ knowing that the Fourier Series coefficients of $y(t)$ are

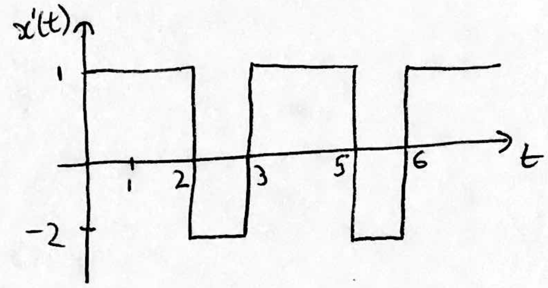
$$c_k = \begin{cases} 1 & k = 0 \\ \frac{3\sin(k2\pi/3)}{2\pi k} e^{-jk2\pi/3} & k \neq 0 \end{cases}$$





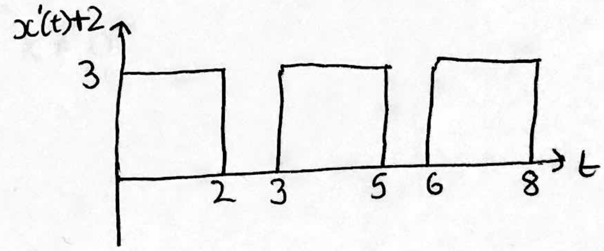
(20 points)

Solution: $x'(t) = \frac{d(x(t))}{dt}$

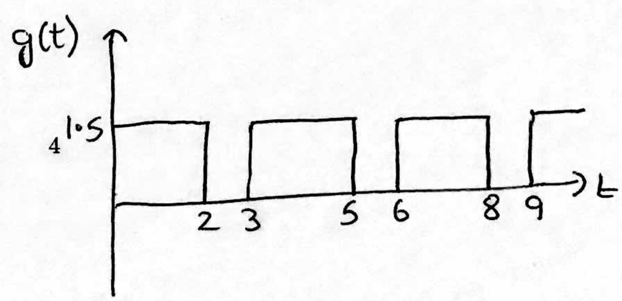


$T = 3\text{sec}$

$\omega_0 = \frac{2\pi}{3}$



Let $g(t) = \frac{x'(t) + 2}{2}$



Note: $g(t) = y(t)$

Thus, $y(t) = \frac{1}{2} x'(t) + 1$

Using the properties of FS,

for $k \neq 0$,

$$c_k = \frac{1}{2}(j\omega_0 k)\, b_k$$

where b_k are the FS coefficients of $x(t)$

$$b_k = \frac{2c_k}{j\omega_0 k} \qquad k \neq 0$$

$$= \frac{2 \times 3\sin(k2\pi/3)\, e^{-jk2\pi/3}}{j\,\frac{2\pi}{3} \times k \times 2\pi k}$$

$$= \frac{9\sin(k2\pi/3)\, e^{-jk2\pi/3}}{j2(\pi k)^2}$$

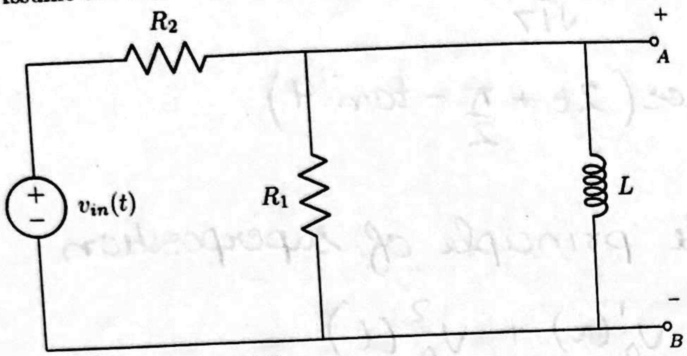
$$b_0 = \frac{1}{T}\int_0^T x(t)\, dt$$

$$= \frac{1}{3} \times 3 = 1$$

$$b_k = \begin{cases} 1 & k = 0 \\ \dfrac{9\sin(2\pi k/3)\, e^{-jk2\pi/3}}{j2(\pi k)^2} & k \neq 0 \end{cases}$$

(Similar to HW1 question 4)

3. Assume the following circuit is at steady state



($R_1 = R_2 = 2\Omega, L = 2H$)

$v_0(t)$ is the output voltage at terminal A and B.

(a) Find the transfer function i.e. $H(\omega) = V_0/V_{in}$ of the circuit at steady state. **(5 points)**

(b) Let $v_{in}(t) = \cos(t) + \cos(2t)$. Find the output voltage $v_0(t)$. **(5 points)**

a) $Z_1 = Z_{R_1} \| Z_L = \dfrac{2 \cdot 2\omega j}{2+2\omega j} = \dfrac{2\omega j}{1+\omega j}$

$V_o = \dfrac{Z_1}{Z_1 + Z_{R_2}} V_{in}$ (voltage divider)

$= \dfrac{2\omega j}{2+2\omega j + 2\omega j} = \dfrac{\omega j}{1+2\omega j} = \dfrac{j\omega}{1+j\omega 2}$

$\Rightarrow H(\omega) = \dfrac{V_o}{V_{in}} = \dfrac{j\omega}{1+j2\omega}$

b) $\omega = 1, 2$

By superposition principle, at $\omega = 1$

$V_{in}' = 1e^{j0}$ (phasor)

$V_o' = H(1) V_{in}'$

$= \dfrac{j}{1+2j} = \dfrac{1}{\sqrt{5}} e^{j(\pi/2 - \tan^{-1}(2))}$

$v_o'(t) = \dfrac{1}{\sqrt{5}} \cos(t + \pi/2 - \tan^{-1}(2))$

$$V_{in}^2 = 1e^{j0} \quad \text{(phasor)} \quad \text{at } \omega = 2$$

$$V_o^2 = H(2)\,V_{in}^2$$

$$= \frac{2j}{1+4j} = \frac{2}{\sqrt{17}} e^{j(\frac{\pi}{2} - \tan^{-1}4)}$$

$$v_o^2(t) = \frac{2}{\sqrt{17}} \cos\left(2t + \frac{\pi}{2} - \tan^{-1}4\right)$$

Hence, by ~~pricipl~~ principle of superposition

$$V_o(t) = v_o^1(t) + v_o^2(t)$$

$$= \frac{1}{\sqrt{5}} \cos\left(t + \frac{\pi}{2} - \tan^{-1}(2)\right) + \frac{2}{\sqrt{17}} \cos\left(2t + \frac{\pi}{2} - \tan^{-1}(4)\right)$$