University of California, San Diego

ECE 45

Midterm - 2

Massimo Franceschetti

Print your name: Solutions

Student ID number:

Note: No books, no notes, no calculators allowed.

Question	Score
1	
2	
3	
Total	

Note: Similar to question 5 HW1.

1. Consider the following input-output relationship expressed in differential form, where $x(t)$ is the input and $y(t)$ is the output of an LTI system

$$\frac{d^3}{dt^3}y(t) + 2\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + y(t) = 6x(t) - 6\frac{d}{dt}x(t)$$

(a) Determine the transfer function i.e. $H(\omega) = Y(\omega)/X(\omega)$ of the LTI system. **(5 points)**

(b) If the input signal is $x(t) = e^{-jt/2+j\pi/2} + e^{jt/2+j3\pi/2}$, then determine the output $y(t)$. **(5 points)**
Note: represent the complex numbers in the polar form in part (b) to receive full credit

a) Equivalent relation in frequency domain is

$$(j\omega)^3 Y(\omega) + 2(j\omega)^2 Y(\omega) + (j\omega) Y(\omega) + Y(\omega) = 6X(\omega) - 6(j\omega)X(\omega)$$

$$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{6-6j\omega}{[(j\omega)^3 + 2(j\omega)^2 + (j\omega) + 1]}$$

$$= \frac{6(1-j\omega)}{1-2\omega^2 + j(\omega-\omega^3)}$$

b) $x(t) = e^{-jt/2 + j\pi/2} + e^{jt/2 + j3\pi/2}$

$$= e^{-jt/2}(j) + e^{jt/2}(-j)$$

$$= (-j)\left(e^{jt/2} - e^{-jt/2}\right) \quad -1)$$

$$\cancel{-j\,\frac{\sin(t/2)}{2j} = \frac{-1}{2}\sin(t/2)}$$

$$= (-j)(2j)\sin(t/2)$$

$$= 2\sin(t/2)$$

$$\left(\begin{aligned} e^{j\pi/2} &= j\sin(\pi/2) = j \\ e^{j3\pi/2} &= j\sin(3\pi/2) = -j \end{aligned}\right)$$

as $\cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$

$\cancel{\omega = 1/2}$

$\cancel{V_{in} = 2e^{-j\pi/2}}$ ~~(phasor)~~

$\omega = 1/2$

$$H(\omega) = H(1/2) = \frac{6(1-j\omega)}{1-\frac{1}{2}+j\left(\frac{1}{2}-\frac{1}{8}\right)}$$

$$= \frac{6(1-j/2)}{1/2 + j3/8}$$

$$= \frac{6\sqrt{\frac{5}{4}}}{(5/8)} e^{-j\left(\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)}$$

$$= \frac{24}{\sqrt{5}} e^{-j\left(\tan^{-1}(1/2)+\tan^{-1}(3/4)\right)}$$

$$H(-1/2) = \frac{24}{\sqrt{5}} e^{j\left(\tan^{-1}(1/2)+\tan^{-1}(3/4)\right)}$$ At $\omega = -1/2$

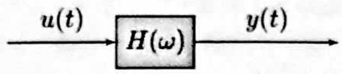
from eq^n 1),

$$y(t) = -j\left(H(1/2)e^{jt/2} - H(-1/2)e^{-jt/2}\right)$$ (as $x(t)$ is in form of Fourier Series)

$$= \frac{48}{\sqrt{5}} \sin\left(\frac{t}{2} - \tan^{-1}(1/2) - \tan^{-1}(3/4)\right)$$

Note: Refer Discussion 7 : Introductory Notes & Example 1

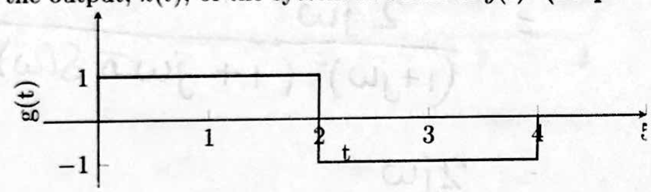
2. Consider the following LTI system with transfer function $H(\omega)$.



If the input to the LTI system is the step function, $u(t)$, let the corresponding output be $y(t) = 2te^{-t}u(t)$.

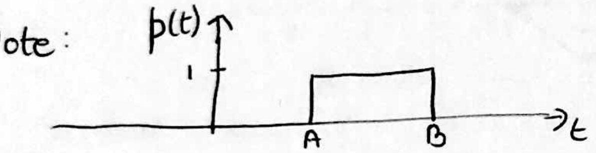
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

(a) Assuming now the input is the signal $g(t)$ indicated below, determine the output, $k(t)$, of the system in terms of $y(t)$. **(15 points)**



(b) Find the transfer function $H(\omega)$ **(5 points)**

a) Note:



$p(t) = u(t-A) - u(t-B)$

(any rectangular wave can be represented using the step function)

Therefore,

$$g(t) = \left(u(t) - u(t-2)\right) - 1\left(u(t-2) - u(t-4)\right)$$

$$= u(t) - 2u(t-2) + u(t-4) \qquad -1)$$

If $u(t)$ is the input, $y(t)$ is ~~the~~ the output of LTI system. Thus,

$$k(t) = y(t) - 2y(t-2) + y(t-4)$$

(from the properties of LTI system)

b)

$$x(t) = u(t)$$

$$X(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$y(t) = 2t e^{-t} u(t)$$

$$Y(\omega) = \frac{2}{(1+j\omega)^2}$$

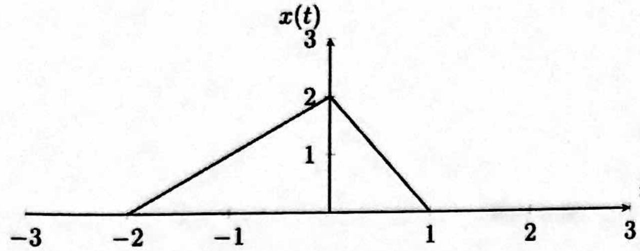
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{(1+j\omega)^2\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)}$$

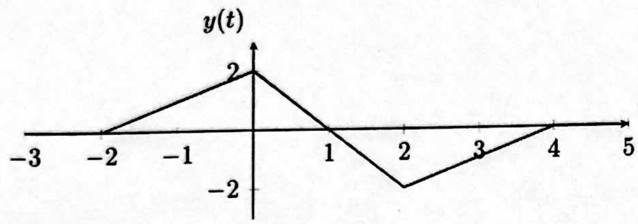
$$= \frac{2j\omega}{(1+j\omega)^2(1+j\omega\pi\delta(\omega))}$$

$$= \frac{2j\omega}{(1+j\omega)^2}$$

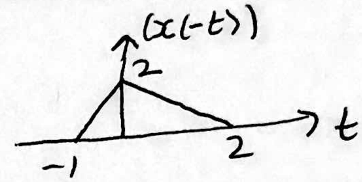
3. **Find the Fourier Transform of the signal $y(t)$ below, knowing that the Fourier Transform of the signal $x(t)$ below is**

$$X(\omega) = \frac{3 - e^{j2\omega} - 2e^{-j\omega}}{\omega^2}$$

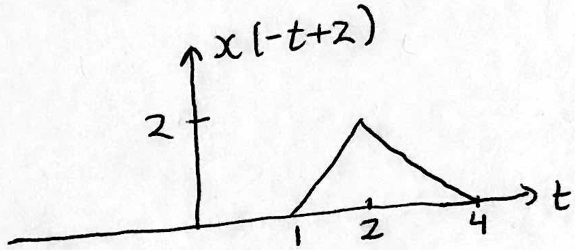




$$y(t) = x(t) - x(-(t-2))$$
$$= x(t) - x(-t+2)$$



$$F(x(-t+2)) = \int_{-\infty}^{\infty} x(-t+2)\, e^{-j\omega t}\, dt$$



Say $t-2 = t'$

$$= \int_{-\infty}^{\infty} x(-t')\, e^{-j\omega(t'+2)}\, dt'$$

$$= e^{-2j\omega} \int_{-\infty}^{\infty} x(-t')\, e^{-j\omega t'}\, dt'$$

$$= e^{-2j\omega} X(-\omega)$$

$$\Rightarrow \quad Y(\omega) = X(\omega) - X(-\omega)\, e^{-2j\omega}$$

$$= \frac{3 - e^{j2\omega} - 2e^{-j\omega} - (3 - e^{-2j\omega} - 2e^{j\omega})\, e^{-2j\omega}}{\omega^2}$$

$$= \frac{3 - e^{j2\omega} - 3e^{-2j\omega} + e^{-j4\omega}}{\omega^2}$$