University of California, San Diego
ECE 45 Spring 2019
MIDTERM EXAM 2

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Print your name:

Student ID Number:

Note: No books, calculators, or other electronic devices allowed.

Question	Score
Problem 1	/15
Problem 2	/10
Problem 3	/15
Total	/40

Signal	Fourier Transform
$x(t)$	$X(j\omega)$
$y(t)$	$Y(j\omega)$
$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
$x^*(t)$	$X^*(-j\omega)$
$x(-t)$	$X(-j\omega)$
$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega)*Y(j\omega)$
$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
$X(t)$	$2\pi x(-j\omega)$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$e^{-at}u(t)$, $\mathrm{Re}\{a\}>0$	$\frac{1}{a+j\omega}$
$\begin{cases}1, & \|t\|\le T\\ 0, & \|t\|>T\end{cases}$	$\frac{2\sin\omega T}{\omega}$
$\frac{\sin Wt}{\pi t}$	$\begin{cases}1, & \|\omega\|\le W\\ 0, & \|\omega\|>W\end{cases}$

Some Useful Formulas:

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}\,d\omega$$

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}\,dt$$

$$u(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases}$$

$$\delta(t)=\frac{d}{dt}u(t)$$

$$e^{j\theta}=\cos\theta+j\sin\theta$$

$$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$\int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2\,d\omega$$

$$e^{j\omega_0 t}\to H(\omega)\to H(\omega_0)e^{j\omega_0 t}$$

1) Your ECE friend did not study very hard at UCSD and designed a strange bandpass filter with the transfer function H(w) is described below:

$$H(w) = G_1(w) + G_1(-w)$$

where $G_1(w) = u(w-(w_1-5)) - 0.5 * u(w-w_1) - 0.5 * u(w-(w_1+5))$

Assume $w_1 = 9$ for all parts.

a) Please graph H(w). (5 Points)

b) Which of the following signals will be “Completely Filtered out”, “Distorted” or “Pass through undistorted” ? Please justify your choice. (2.5 Points each)

i) $x_1(t) = sinc(w_1 t)$

ii) $x_2(t) = cos(8t) + sin(8t)$

iii) $x_3(t) = j$

iv) $x_4(t) = \delta(t)$

2) Find the frequency response, H(w), for the following expression: (10 points)

$$b \cdot y(t) - c \cdot \frac{d(y(t))}{dt} - \frac{d^2(y(t))}{dt^2} = x(t) + \frac{d(x(t))}{dt}$$

where b=4, c=3.

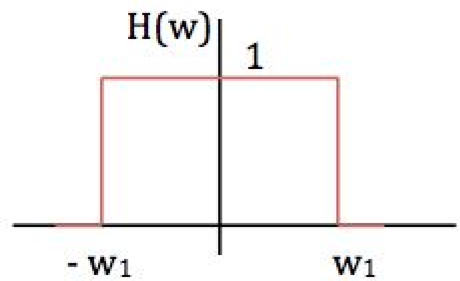
3) You have a system $x(t) \rightarrow H(w) \rightarrow y(t)$. Your input $x(t)$ is composed of two functions multiplied by cosines as seen below:

$$x(t) \;=\; cos(at) * f(t) \;+\; cos(bt) * g(t)$$

The fourier transforms of $f(t) \rightarrow F(w)$ and $g(t) \rightarrow G(w)$ are defined below:

$$G(w) = \begin{cases} 1-|w| & -1 \le w \le 1 \\ 0 & else \end{cases} \qquad\qquad F(w) = \begin{cases} 1 & -1 \le w \le 1 \\ 0 & else \end{cases}$$

H(w) is given below:



Assume w_1 = 5, a = 3, b = 7 for all parts

a) Draw X(w) in the frequency domain. (4 Points)

b) Find Y(w). (6 Points)

c) Find y(t). (5 Points)