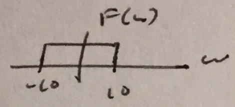
① $\omega_s = 50\,Hz$, $\omega_s > 2\omega_m$ $\omega_m < 25\,Hz$

ⓐ $f(t) = \mathrm{sinc}(10t) \longleftrightarrow$



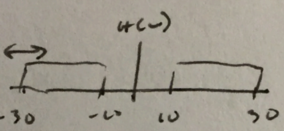
$\omega_m = 10\,Hz$ <u>Yes</u>

ⓑ $g(t) = \sin^2(20t) \longleftrightarrow \mathcal{F}(\sin(20t)) * \mathcal{F}(\sin(20t))$

$\mathcal{F}(\sin(20t)) = \frac{\pi}{j}[\delta(\omega-20) - \delta(\omega+20)]$

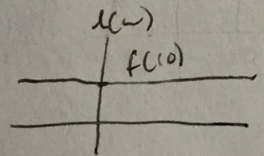
$\omega_m = 40$ <u>No</u>

ⓒ $h(t) = f(t) \cdot \cos(20t) \longleftrightarrow$



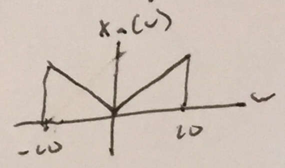
$\omega_m = 30$ <u>No</u>

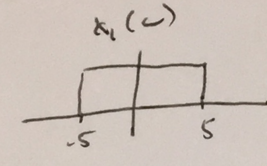
ⓓ $l(t) = f(t) \cdot \delta(t-10) = f(10)\,\delta(t-10)$



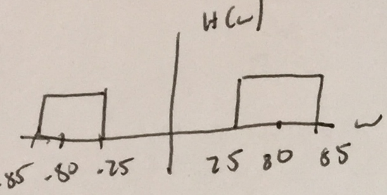
<u>No</u>

2 $\omega_a = 100$ $\omega_1 = 80$ $x_r(t) = x_1(t)\cos(\omega_1 t) + x_a(t)\cos(\omega_a t)$





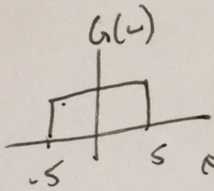
a)



Filters out frequencies > 85 which is where AJ Networks is transmitting. It's purpose is to filter out all other undesired signals

b) $f(t) = \cos\omega_1 t$, Demodulate the signal.
must match the carrier frequency.

c)

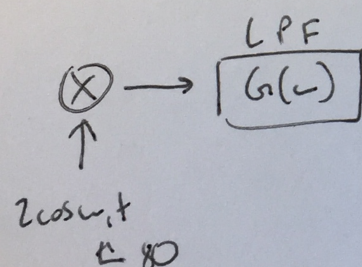


Filter out double frequency term from demodulation

← Bandwidth of desired signal.

d) I would change the modulation frequency ω_1 to in the range 1005 - 1095.

e)

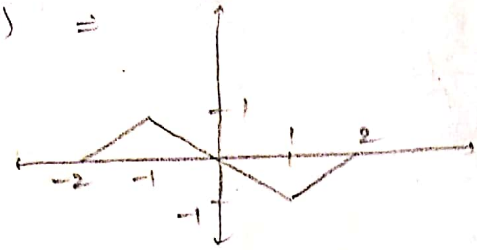


$$G(\omega) = \begin{cases} 1 & -10 \le \omega \le 10 \\ 0 & \text{else} \end{cases}$$

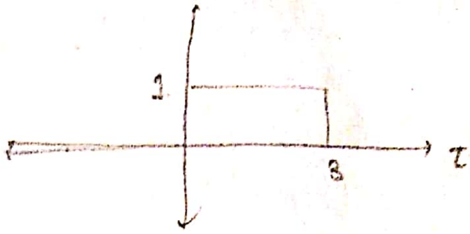
$$\cos\omega_1 t \cdot \cos\omega_a t = \frac{1}{2}\left[\cos(\omega_1 + \omega_a)t + \cos(\omega_1 - \omega_a)t\right]$$

$G(\omega)$ must be small enough to filter out both these frequency terms, but must also allow all of $X_1(\omega)$ to go through.

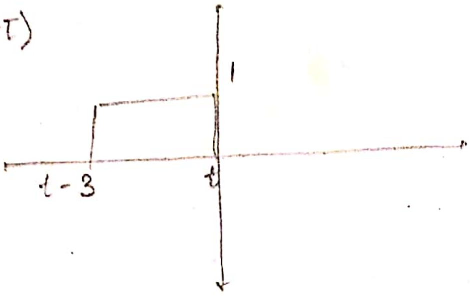
3. $h(t) =$



$g(\tau)$



$g(t-\tau)$



Case I : $t < -2$ No overlap. $y(t) = 0$

Case II : $-2 < t < -1$

$$y(t) = \int_{-2}^{t} (\tau + 2)\, d\tau = \frac{\tau^2}{2} + 2\tau \Big|_{-2}^{t}$$

$$= \frac{t^2}{2} + 2t - 2 + 4 = \frac{t^2}{2} + 2t + 2$$

Case III : $-1 < t < 0$

$$y(t) = \int_{-2}^{-1} (\tau+2)\, d\tau + \int_{-1}^{t} -\tau\, d\tau$$

$$= \frac{\tau^2}{2} + 2\tau \Big|_{-2}^{-1} - \frac{\tau^2}{2}\Big|_{-1}^{t}$$

$$= \frac{1}{2} - 2 - 2 + 4 - \frac{t^2}{2} + \frac{1}{2} = 1 - t^2/2$$

Case IV : $0 < t < 1$

$$y(t) = \int_{-2}^{-1} (\tau+2)\, d\tau + \int_{-1}^{0} -\tau\, d\tau + \int_{0}^{t} -\tau\, d\tau$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{t^2}{2} = 1 - \frac{t^2}{2}.$$

Case V $1 < t < 2.$

$$y(t) = \int_{t-3}^{-1} (\tau+2)\, d\tau + \int_{-1}^{0} -\tau\, d\tau + \int_{0}^{1} -\tau\, d\tau + \int_{1}^{t} (\tau - 2)\, d\tau$$

$$= \frac{\tau^2}{2} + 2\tau \Big|_{t-3}^{-1} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{\tau^2}{2} - 2\tau \Big|_{1}^{t}$$

$$= \left(\frac{1}{2} - 2\right) - \left(\frac{(t-3)^2}{2} + 2(t-3)\right) + \frac{t^2}{2} - 2t - \frac{1}{2} + 2$$

$$= \frac{t^2}{2} - 2t - \left[\frac{(t-3)^2}{2} + 2(t-3)\right]$$

Case VI : $2 < t < 3$

$$y(t) = \int_{t-3}^{0} -\tau\, d\tau + \int_{0}^{1} -\tau\, d\tau + \int_{1}^{2} (\tau - 2)\, d\tau$$

$$= \frac{(t-3)^2}{2} - 1.$$

Case VII : $3 < t < 4$

$$y(t) = \int_{t-3}^{1} -\tau\, d\tau + \int_{1}^{2} (\tau - 2)\, d\tau$$

$$= -\frac{\tau^2}{2}\Big|_{t-3}^{1} - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{(t-3)^2}{2} - \frac{1}{2} = \frac{(t-3)^2}{2} - 1$$

Case VIII : $4 < t < 5$

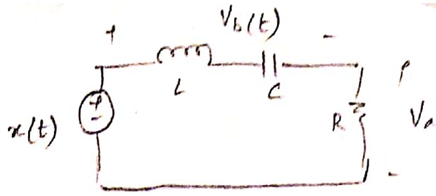
$$y(t) = \int_{t-3}^{2} (\tau - 2)\, d\tau$$

$$= \frac{\tau^2}{2} - 2\tau \Big|_{t-3}^{2}$$

$$= -2 - \frac{(t-3)^2}{2} + 2(t-3)$$

Case VIII : $t > 5$ No overlap $y(t) = 0$.

4. a)



$$V_a(j\omega) = \frac{R}{\left(j\omega L + \frac{1}{j\omega C} + R\right)} \cdot X(j\omega)$$

$$H_1(j\omega) = \frac{V_a(j\omega)}{X(j\omega)} = \frac{jR\omega C}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{3j\omega}{1 + 3j\omega - 2\omega^2}$$

Similarly, $$V_b(j\omega) = \frac{j\omega L + \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C} + R} \cdot X(j\omega)$$

$$H_2(j\omega) = \frac{V_b(j\omega)}{X(j\omega)} = \frac{1 - \omega^2 LC}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{1 - 2\omega^2}{1 + 3j\omega - 2\omega^2}.$$

b) $$X(j\omega) = \frac{1}{3 + j\omega}.$$

$$V_a(j\omega) = \frac{3j\omega}{(1 + 3j\omega - 2\omega^2)} \cdot \frac{1}{(3 + j\omega)}$$

$$= \frac{3j\omega}{(1 + j\omega)(1 + 2j\omega)(3 + j\omega)} = \frac{A}{(1 + j\omega)} + \frac{B}{(1 + 2j\omega)} + \frac{C}{(3 + j\omega)}$$

Using partial fractions, we find

$$A = V_a(j\omega)(1 + j\omega)\Big|_{(j\omega + 1) = 0} = \frac{(-3)}{(-1)(2)} = 3/2.$$

$$B = V_a(j\omega)(1 + 2j\omega)\Big|_{(1 + 2j\omega) = 0} = \frac{3(-1/2)}{(1/2)(5/2)} = -6/5$$

$$C = V_a(j\omega)(3 + j\omega)\Big|_{(3 + j\omega) = 0} = \frac{3(-3)}{(-2)(-5)} = -9/10$$

$$V_a(j\omega) = \frac{3}{2}\left(\frac{1}{1 + j\omega}\right) - \frac{6}{5}\left(\frac{1}{1 + 2j\omega}\right) - \frac{9}{10}\left(\frac{1}{3 + j\omega}\right)$$

⑤ (a) $g(t) = x(-2t-4)$, given X_n, find G_n, frequency ω_0 for X_n

Solve

1) $h(t) = x(t-4)$, $H_n = X_n e^{-j\omega_0 4n}$

2) $g(t) = h(-2t) = x(-2t-4)$, $G_n = H_{-n} = \underline{X_{-n} e^{j4\omega_0 n}}$, frequency $2\omega_0$

$\uparrow$

extra credit +1

⑥ $h(t) = \frac{d}{dt}(y(t-1))$, given Y_n, find H_n

1) $k(t) = y(t-1)$, $K_n = Y_n e^{-j\omega_0 \cdot 1 n}$

2) $h(t) = \frac{d}{dt} k(t)$, $H_n = j\omega_0 n K_n = \underline{j\omega_0 n Y_n e^{-j\omega_0 n}}$

Taking inverse fourier transform gives us.

$$v_a(t) = \left[\frac{9}{2}e^{-t} - \frac{3}{5}e^{-t/2} - \frac{9}{10}e^{-3t}\right]u(t).$$

5. c) $$x(t) = \int_{6}^{\infty}(\tau^2+5)\,\delta(t-5)\,d\tau$$

$$= 30\int_{6}^{\infty}\delta(t-5)\,d\tau = 0.$$

d). $$x(t) = \frac{2}{jt+4}\,u(t).$$

Method 1 : Solving in time domain

$$E = \int_{-\infty}^{\infty}|x^2(t)|\,dt = \int_{0}^{\infty}\frac{4}{16+t^2}\cdot dt = \frac{1}{4}4\tan^{-1}\left(\frac{t}{4}\right)\Big|_0^{\infty} = \frac{\pi}{2}$$

Method 2 : Solving in frequency domain.

Using Parseval's theorem,

$$E = \int_{-\infty}^{\infty}|x^2(t)|\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(w)|^2\cdot dw$$

Using the property of duality,

$$X(w) = 2\pi\left[2\cdot e^{4w}\right]u(-w).$$

$$E = \frac{1}{2\pi}\int_{-\infty}^{0}4\pi^2\times 4\times e^{8w}\cdot dw$$

$$= \pi/2$$