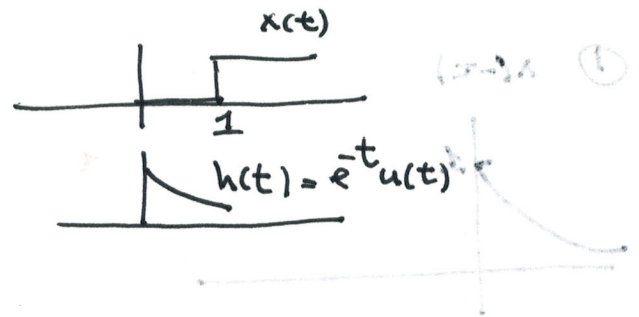
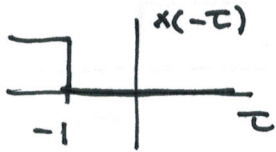


CONVOLUTION

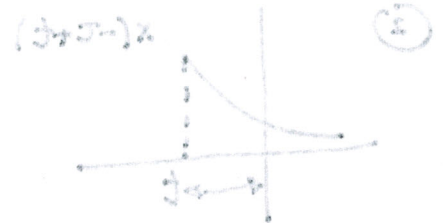
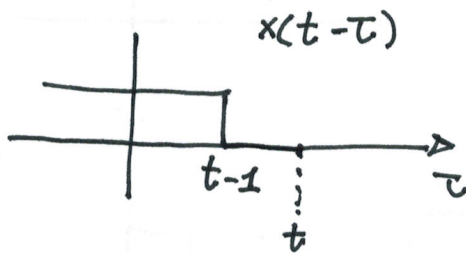
$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$x(t) = u(t)$
 $h(t) = e^{-t} u(t)$

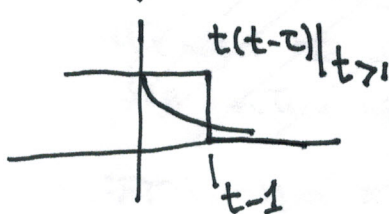
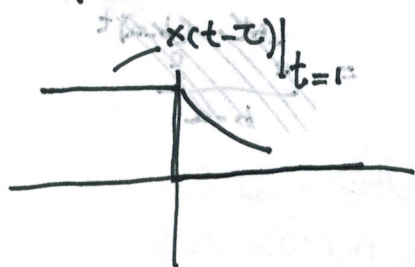
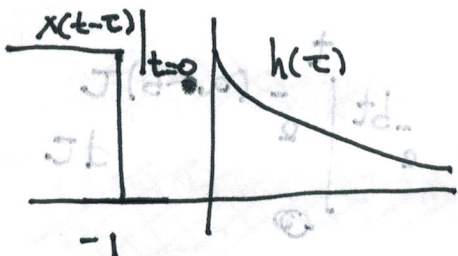
① Flipping



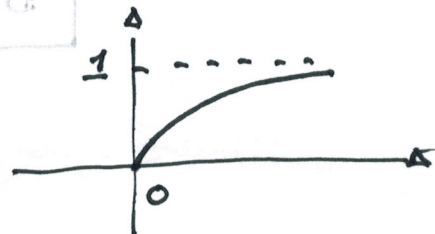
② Shifting



③ Evaluation



$$y(t) = \begin{cases} 0 & |t < 1| \\ \int_0^{t-1} e^{-\tau} d\tau & |t > 1| \end{cases} = \begin{cases} 0 & |t < 1| \\ 1 - e^{-(t-1)} & |t > 1| \end{cases} u(t-1)$$

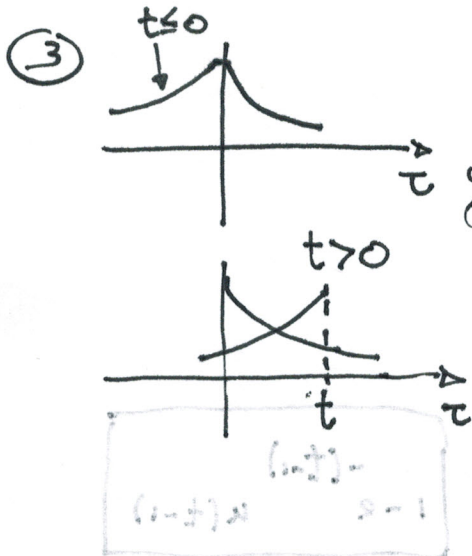
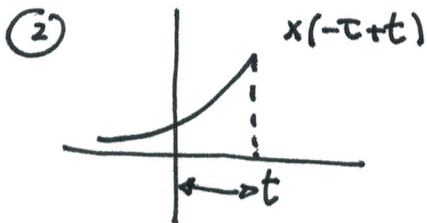
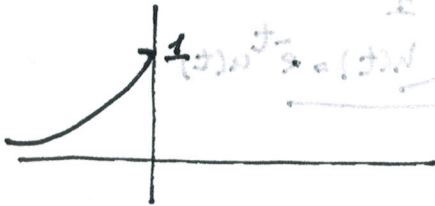


$$x(t) = e^{-at} u(t)$$

$$h(t) = e^{-bt} u(t)$$

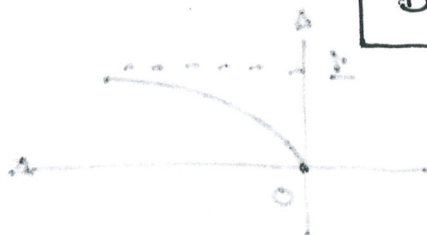
$$\int_{-\infty}^{\infty} b(\sigma - t) \delta(\sigma) x(\sigma) d\sigma = \int_{-\infty}^{\infty} b(\sigma) \delta(\sigma - t) x(\sigma) d\sigma = b(t) x(t)$$

① $x(-\tau)$

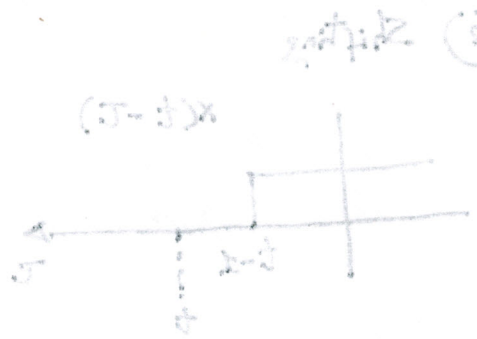
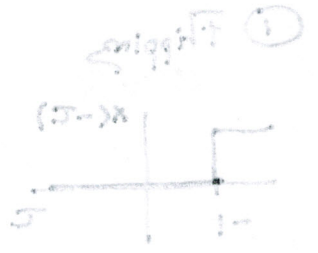


$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t e^{-a\tau} e^{-b(t-\tau)} d\tau & t > 0 \end{cases}$$

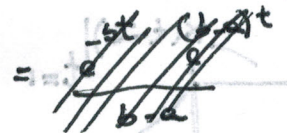
$$= \frac{e^{-at} - e^{-bt}}{b - a} u(t)$$



Convolution



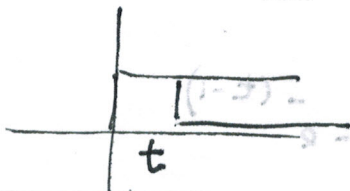
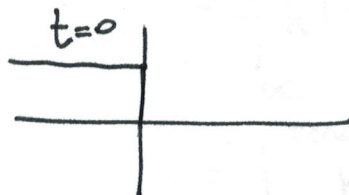
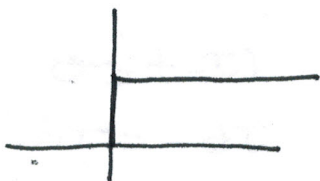
$$\int_0^t e^{-bt} e^{-(a-b)\tau} d\tau = e^{-bt} \left[\frac{e^{-(a-b)\tau}}{-(a-b)} \right]_0^t = \frac{e^{-bt} (1 - e^{-(a-b)t})}{b - a}$$



convolve Unit Steps

$$x(t) = u(t)$$

$$h(t) = u(t)$$

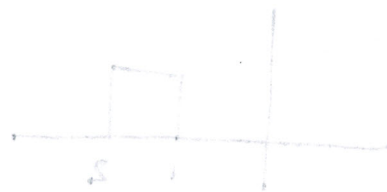


$$y(t) = 0 \quad t < 0$$

$$y(t) = \int_0^t d\tau = t$$

$$y(t) = t(u(t))$$

$$(x)u(t) = x(t)u(t)$$

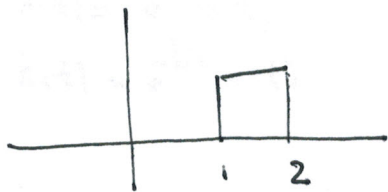


$$0 = (x)u(t) \quad t > 0$$

$$(x)u(t) = x(t)u(t)$$

$$x(t)u(t) = x(t)u(t)$$

$$x(t) =$$

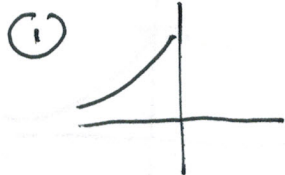


$$h(t) = \frac{-t}{e} u(t)$$



$$299t^2 \sin U \sin v m$$

$$(t) * = (t) * \\ (t) * = (t) *$$

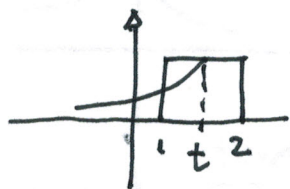


③ $t < 1 \quad y(t) = 0$

$$1 \leq t \leq 2$$

$$\int_1^t e^{-(t-\tau)} d\tau$$

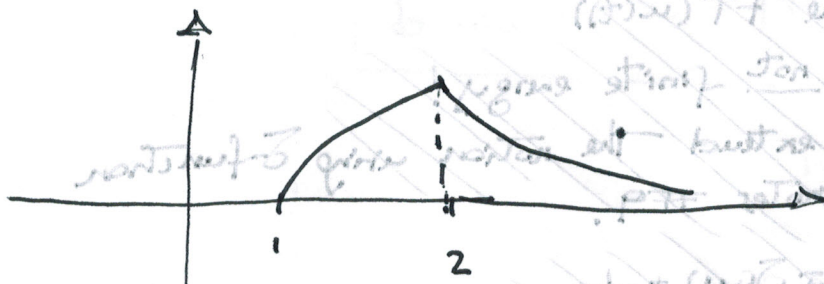
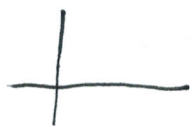
$$= \left[-e^{-(t-\tau)} \right]_1^t = 1 - e^{-(t-1)}$$



$$t \geq 2$$

$$\int_1^2 e^{-(t-\tau)} d\tau$$

$$= \left[-e^{-(t-\tau)} \right]_1^2 = e^{-(t-2)} - e^{-(t-1)}$$



$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

$$(t) * = (t) *$$

Convolution is commutative

$$h * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$\sigma = t - \tau$$

$$d\sigma = -d\tau$$

$$= - \int_{\infty}^{-\infty} h(t-\sigma) x(\sigma) d\sigma$$

$$= \int_{-\infty}^{\infty} h(t-\sigma) x(\sigma) d\sigma$$