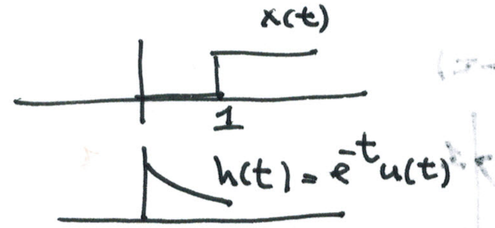
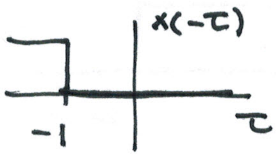
CONVOLUTION

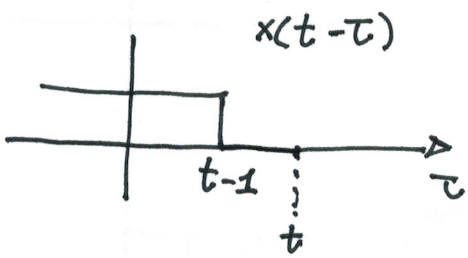
$$y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$



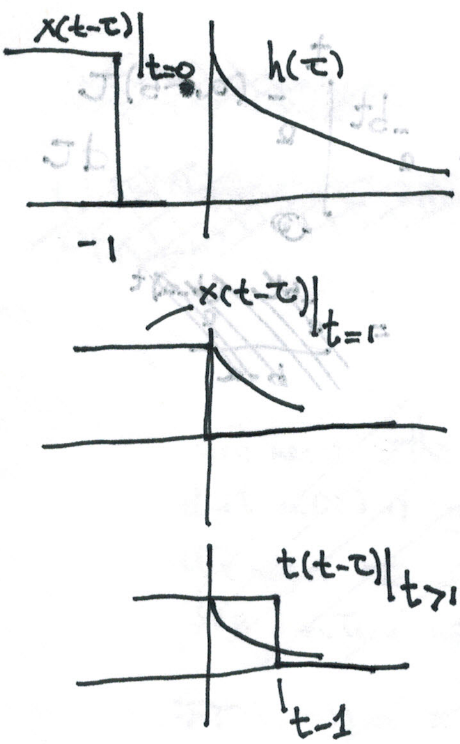
① Flipping



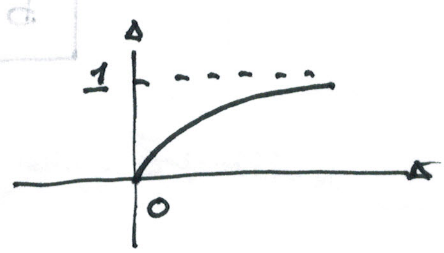
② Shifting



③ Evaluation



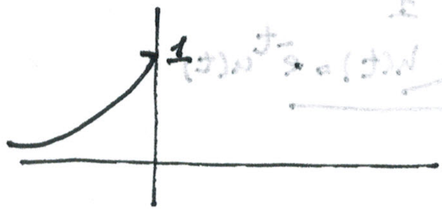
$$y(t) = \begin{cases} 0 & \boxed{t<1} \\ \displaystyle\int_0^{t-1} e^{-\tau}\,d\tau & \boxed{t>1} \end{cases} = \boxed{1 - e^{-(t-1)}\,u(t-1)}$$



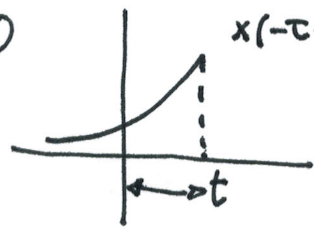
$x(t) = e^{-at}\,u(t)$

$h(t) = e^{-bt}\,u(t)$

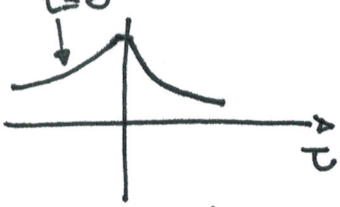
① $x(-\tau)$



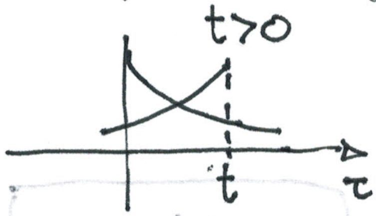
② $x(-\tau+t)$



③ $t \le 0$



$t > 0$

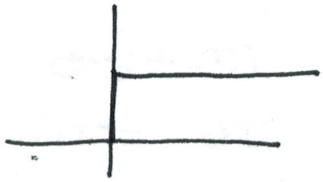


$$y(t) = \begin{cases} 0 & t < 0 \\ \displaystyle\int_0^t e^{-a\tau}\, e^{-b(t-\tau)}\, d\tau = e^{-bt} \int_0^t e^{-(a-b)\tau}\, d\tau \end{cases}$$

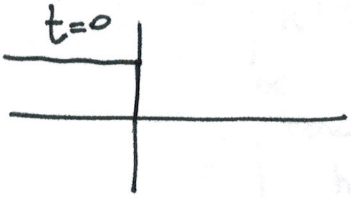
$$= \boxed{\frac{e^{-at} - e^{-bt}}{b-a}\, u(t)}$$

Convolve Unit Steps

$x(t) = u(t)$
$h(t) = u(t)$

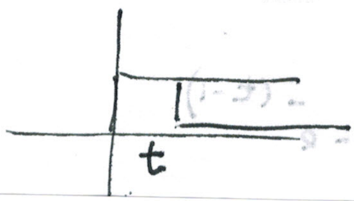


t=0



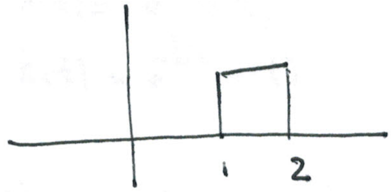
$y(t) = 0 \quad t<0$

$$y(t) = \int_0^t d\tau = t$$

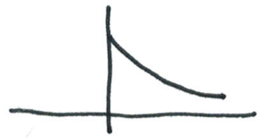


$$\boxed{y(t) = t(u(t))}$$

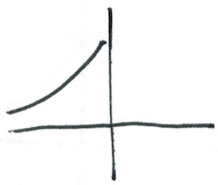
$x(t) =$



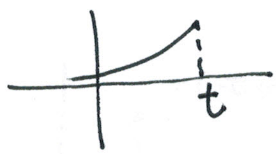
$h(t) = e^{-t} u(t)$



①

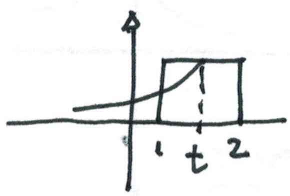


②

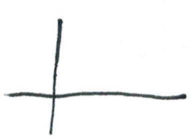


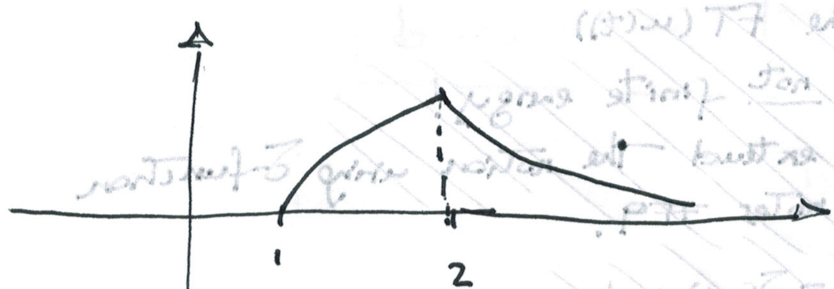
③ $t<1 \quad y(t)=0$

$\boxed{1 \leq t \leq 2}$ $\displaystyle\int_1^t e^{-(t-\tau)}\,d\tau = \Big[e^{-(t-\tau)}\Big]_1^t = 1 - e^{-(t-1)}$



$\boxed{t \geq 2}$ $\displaystyle\int_1^2 e^{-(t-\tau)}\,dt = \Big[e^{-(t-\tau)}\Big]_1^2 = e^{-(t-2)} - e^{-(t-1)}$





Convolution is commutative

$$h*x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$\sigma = t-\tau)$

$d\sigma = -d\tau$

$$= -\int_{\infty}^{-\infty} h(t-\sigma)\, x(\sigma)\, d\sigma$$

$$= \int_{-\infty}^{+\infty} h(t-\sigma)\, x(\sigma)\, d\sigma$$