# University of California, San Diego
# ECE 45
# Midterm Exam

Massimo Franceschetti

**Print your name :** _____
**Student ID Number :** _____

- No Books, No Notes, No calculators allowed

| Question | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

## Some Useful Formulas:

$$Ae^{j\theta} = A\cos(\theta) + jA\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad , \quad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \quad , \quad C_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt$$



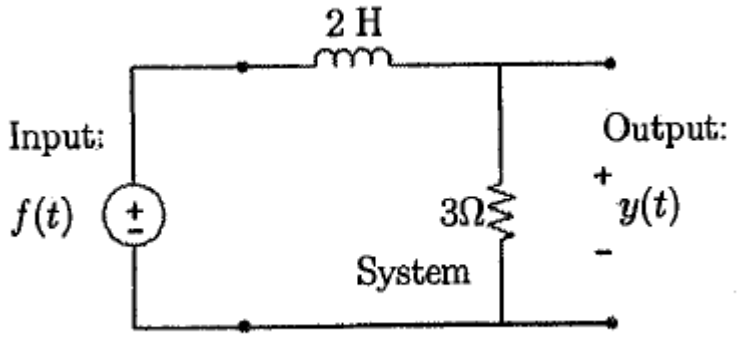
**TABLE 3.1** PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

| Property | Section | Periodic Signal | Fourier Series Coefficients |
|---|---|---|---|
| | | $x(t)$, $y(t)$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$ | $a_k$, $b_k$ |
| Linearity | 3.5.1 | $Ax(t) + By(t)$ | $Aa_k + Bb_k$ |
| Time Shifting | 3.5.2 | $x(t - t_0)$ | $a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$ |
| Frequency Shifting | | $e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$ | $a_{k-M}$ |
| Conjugation | 3.5.6 | $x^*(t)$ | $a^*_{-k}$ |
| Time Reversal | 3.5.3 | $x(-t)$ | $a_{-k}$ |
| Time Scaling | 3.5.4 | $x(\alpha t)$, $\alpha > 0$ (periodic with period $T/\alpha$) | $a_k$ |
| Periodic Convolution | | $\int_T x(\tau)y(t-\tau)d\tau$ | $Ta_k b_k$ |
| Multiplication | 3.5.5 | $x(t)y(t)$ | $\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$ |
| Differentiation | | $\frac{dx(t)}{dt}$ | $jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$ |
| Integration | | $\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$) | $\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$ |
| Conjugate Symmetry for Real Signals | 3.5.6 | $x(t)$ real | $a_k = a^*_{-k}$; $\mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\}$; $\mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\}$; $\lvert a_k\rvert = \lvert a_{-k}\rvert$; $\measuredangle a_k = -\measuredangle a_{-k}$ |
| Real and Even Signals | 3.5.6 | $x(t)$ real and even | $a_k$ real and even |
| Real and Odd Signals | 3.5.6 | $x(t)$ real and odd | $a_k$ purely imaginary and odd |
| Even-Odd Decomposition of Real Signals | | $x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real]; $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real] | $\mathcal{R}e\{a_k\}$; $j\mathcal{I}m\{a_k\}$ |

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

1.) Determine the frequency response of the circuit shown below




And use it to determine the output y(t) of the system if it is subjected to an input f(t) shown below

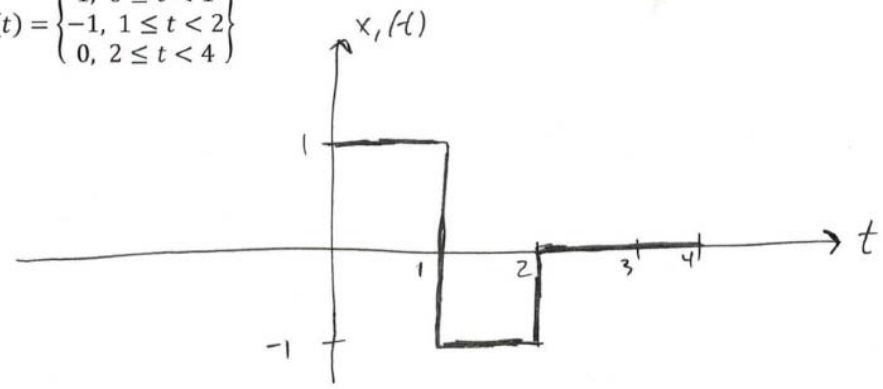
$$f(t) = 5 + \sin(2t) + \cos(4t)$$

2.) Draw the Bode diagrams (amplitude and phase) for the following frequency response:

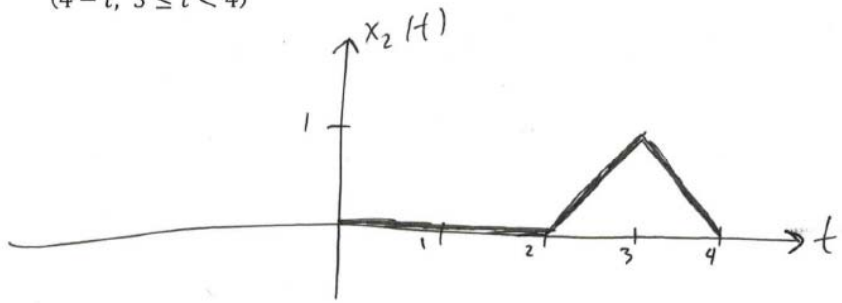
$$H(\omega) = 10^6 \frac{(j\omega)^2 + j\omega}{(j\omega + 10)(j\omega + 10)(j\omega + 100)}$$

3.) Fourier Series: Determine the Fourier series representations of the following periodic signals, each described below over one period.

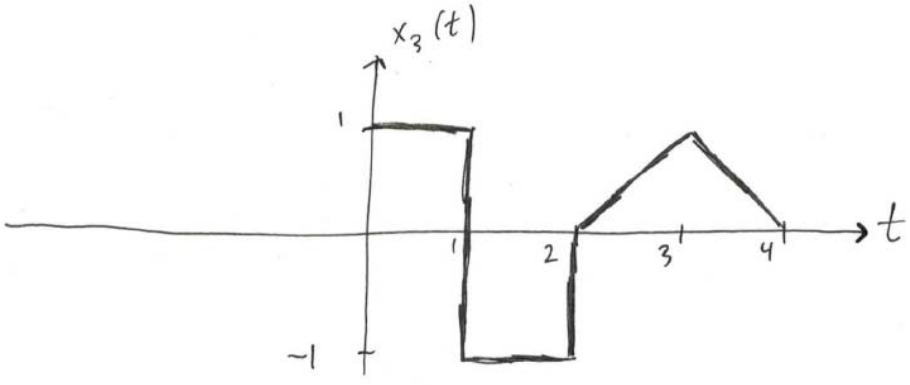
(a) $x_1(t) = \begin{Bmatrix} 1, & 0 \le t < 1 \\ -1, & 1 \le t < 2 \\ 0, & 2 \le t < 4 \end{Bmatrix}$




(b) $x_2(t) = \begin{Bmatrix} 0, & 0 \le t < 2 \\ t-2, & 2 \le t < 3 \\ 4-t, & 3 \le t < 4 \end{Bmatrix}$




(c) $x_3(t) = \begin{Bmatrix} 1, & 0 \le t < 1 \\ -1, & 1 \le t < 2 \\ t-2, & 2 \le t < 3 \\ 4-t, & 3 \le t < 4 \end{Bmatrix}$



4.) Let x(t) be a periodic signal whose Fourier series coefficients are

$$X_k = \begin{cases} 2, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|} & \end{cases}$$

Use Fourier series properties to answer the following questions:

(a) Is x(t) real?
(b) Is x(t) even?
(c) Is dx(t)/dt even?