ECE 45 Homework 1 Solutions

Problem 1.1 Find the magnitude and phase of the following complex number

$$\frac{(1-j)\,(2e^{-j\pi/3})\,(\sin(1))\,(j^2)}{(2+2j)\,(-j\cos(1))\,(e^{j\pi})}$$

without using a calculator. Simplify as much as possible.

Solution:

Recall when taking the product of complex numbers we have $XY = |X|e^{j\angle X}\,|Y|e^{j\angle Y}$, so the magnitude of the product is $|X||Y|$ and the phase of the product is $\angle X + \angle Y$. Thus

$$\begin{aligned}\left|\frac{(1-j)\,(2e^{-j\pi/3})\,(\sin(1))\,(j^2)}{(2+2j)\,(-j\cos(1))\,(e^{j\pi})}\right| &= \frac{|1-j|\,|2e^{-j\pi/3}|\,|\sin(1)|\,|j^2|}{|2+2j|\,|-j\cos(1)|\,|e^{j\pi}|} \\ &= \frac{(\sqrt{2})\,(2)\,(\sin(1))\,1}{(\sqrt{8})\,(\cos(1))\,1} \\ &= \frac{\sin(1)}{\cos(1)} = \tan(1)\end{aligned}$$

and

$$\begin{aligned}\angle\left(\frac{(1-j)\,(2e^{-j\pi/3})\,(\cos(1))\,(j^2)}{(2+2j)\,(-j\sin(1))\,(e^{j\pi})}\right) &= \angle(1-j) + \angle(2e^{-j\pi/3}) + \angle(\cos(1)) + \angle(j^2) \\ &\quad - \angle(2+2j) - \angle(-j\sin(1)) - \angle(e^{j\pi}) \\ &= (-\pi/4) + (-\pi/3) + (0) + (\pi) - (\pi/4) - (-\pi/2) - (\pi) \\ &= -\pi/3.\end{aligned}$$

Thus the above complex number can be written as:

$$\tan(1)e^{-j\pi/3} = \frac{\tan(1)}{2}\,(1-\sqrt{3}).$$

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Problem 1.2 Let $X = 2 + j + 2e^{-j2\pi/3} - e^{j\pi/2}$.

(a) Find the real portion of X.

(b) Find the phase of the complex conjugate of X.

(c) Let $Y = -2 + 2j$. Plot X/Y on the complex plane (real and imaginary axes).

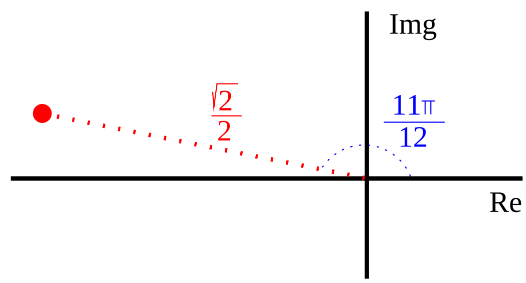
Solution:

(a) $X = 2 + j + 2\,(-1/2 - j\,\sqrt{3}/2) - j = 1 - j\sqrt{3} \;\Rightarrow\; \mathrm{Real}(X) = 1$.

(b) $X = 1 - j\sqrt{3} \;\Rightarrow\; X^* = 1 + j\sqrt{3} = 2e^{j\pi/3} \;\Rightarrow \angle X^* = \pi/3$.

(c) $X = 1 - j\sqrt{3} = 2e^{-j\pi/3}$ and $Y = -2 + 2j = 2\sqrt{2}e^{j3\pi/4}$. Thus

$$\frac{X}{Y} = \frac{2\,e^{-j\pi/3}}{2\,\sqrt{2}\,e^{j3\pi/4}} = \frac{\sqrt{2}}{2}\,e^{j11\pi/12}.$$



Problem 1.3 Let $f(x) = \begin{cases} 1 & \text{if } -1 \le x \le 0 \\ -1 & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $f(2x+1)$.

(b) Plot the magnitude of $e^{jx}\,f(x)$.

(c) If the *energy* of a signal $g(x)$ is defined to be

$$E[g(x)] = \int_{-\infty}^{\infty} |g(x)|^2\,dx$$

how does the energy of $g(x)$ compare to the energy of $e^{jx}\,g(x)$?

Solution:

(a) We have

$$f(2x+1) = \begin{cases} 1 & \text{if } -1 \le 2x+1 \le 0 \\ -1 & \text{if } 0 < 2x+1 < 2 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1 & \text{if } -1 \le x \le -1/2 \\ -1 & \text{if } -1/2 < x < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

(b) $|e^{jx}| = 1$ for all x, so

$$|e^{jx}\,f(x)| = |e^{jx}|\,|f(x)| = |f(x)| = \begin{cases} 1 & \text{if } -1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(c) Since $|e^{jx}| = 1$, for all x, we have

$$E[g(x)] = \int_{-\infty}^{\infty} |g(x)|^2 \, dx = \int_{-\infty}^{\infty} |e^{jx}|^2 |g(x)|^2 \, dx = \int_{-\infty}^{\infty} |e^{jx} g(x)|^2 \, dx E \left[e^{jx} g(x) \right]$$

Problem 1.4 Represent the following sinusoidal functions as phasors.

(a) $f_1(t) = 3\cos(4t) - 4\sin(4t)$

(b) $f_2(t) = 2(\cos(\omega t) + \cos(\omega t + \pi/4))$

(c) $f_3(t) = \cos^2(t) - \sin^2(t)$

Solution:

(a) Taking the phasor transform of $f_1(t)$ with frequency 4 yields:

$$F_1 = 3 - 4e^{-j\pi/2} = 3 + 4j.$$

(b) Taking the phasor transform of $f_2(t)$ with frequency ω yields:

$$F_2 = 2(1 + e^{j\pi/4}).$$

(c) We need to first verify $f_4(t)$ has a single frequency component. Recall

$$\cos^2(t) = \left(\frac{e^{jt} + e^{-jt}}{2} \right)^2 = \cdots = \frac{1 + \cos(2t)}{2}$$
$$\sin^2(t) = \left(\frac{e^{jt} - e^{-jt}}{2j} \right)^2 = \cdots = \frac{1 - \cos(2t)}{2}$$

and so $f_3(t) = \cos(2t)$. Taking the phasor transform of $f_3(t)$ with frequency 2 yields $F_3 = 1$.

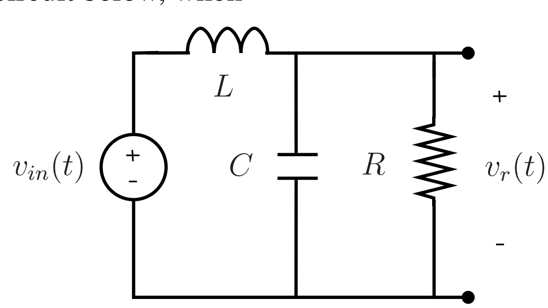
Problem 1.5 Find the voltage $v_r(t)$ in the circuit below, when

(a) $v_{in}(t) = 1/3$

(b) $v_{in}(t) = \sin(t)$

(c) $v_{in}(t) = 1 + 2\sin(t - \pi)$.

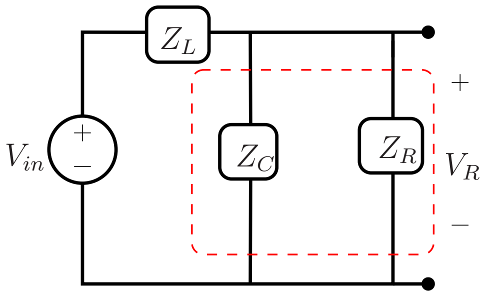
where $R = 1\Omega$, $C = 1/2F$, $L = 2H$.



Solution:

We need to find the transfer function $H(\omega) = V_r/I_{in}$.

Assume that $i_{in}(t)$ is sinusoidal with frequency ω. Then by taking the phasor transformation of the circuit, we have



Voltage divider:

$$V_r = V_{in}\frac{Z_C//Z_R}{Z_C//Z_R + Z_L}.$$

where $Z_R = 2$, $Z_C = \frac{2}{j\omega}$, and $Z_L = 2j\omega$.

And so

$$H(\omega) = \frac{1}{1 + Z_L/Z_R + Z_L/Z_C} = \frac{1}{1 + 2j\omega + (j\omega)^2} = \frac{1}{(j\omega + 1)^2}.$$

(a) In this case, we have $\omega = 0$ and $V_{in} = 1/3$, so

$$V_r = H(0)V_{in} = 1/3 \longrightarrow v_r(t) = 1/3.$$

(b) In this case, we have $\omega = 1$ and $V_{in} = e^{-j\pi/2}$, so

$$V_r = H(1)V_{in} = \frac{e^{-j\pi/2}}{\left(\sqrt{2}e^{j\pi 4}\right)^2} = -\frac{1}{2} \longrightarrow v_r(t) = -\frac{1}{2}\cos(t).$$

(c) Here we utilize the fact an RLC circuit is linear (i.e. super-position) and time-invariant.

Let $v_{in}^{(1)}(t) = 1/3$ and $v_{in}^{(2)}(t) = \sin(t)$ and for each $k = 1, 2$, let $v_r^{(k)}(t)$ be the voltage across the resistor when $v_{in}^{(k)}(t)$ is the input voltage.

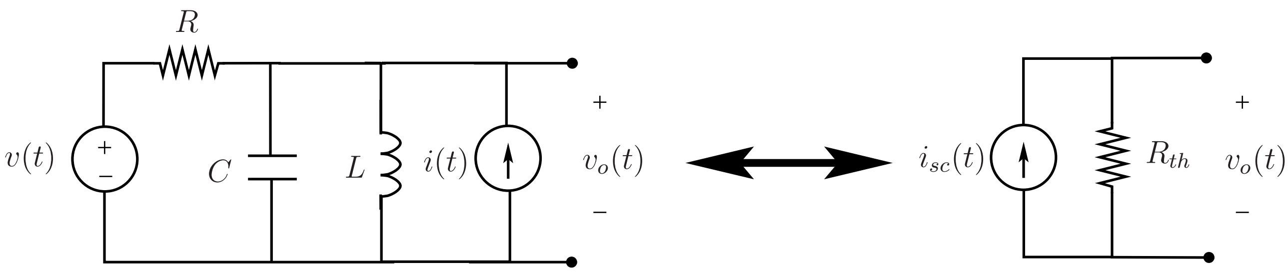
Then by parts (a) and (b), we have $v_r^{(1)}(t) = 1/3$ and $v_r^{(2)}(t) = -\frac{1}{2}\cos(t)$.

We have $v_{in}(t) = 3v_{in}^{(1)}(t) + 2v_{in}^{(2)}(t - \pi)$, so by linearity and time invariance, we have

$$v_r(t) = 3v_r^{(1)}(t) + 2v_r^{(2)}(t - \pi) = 1 - \cos(t - \pi) = 1 + \cos(t).$$

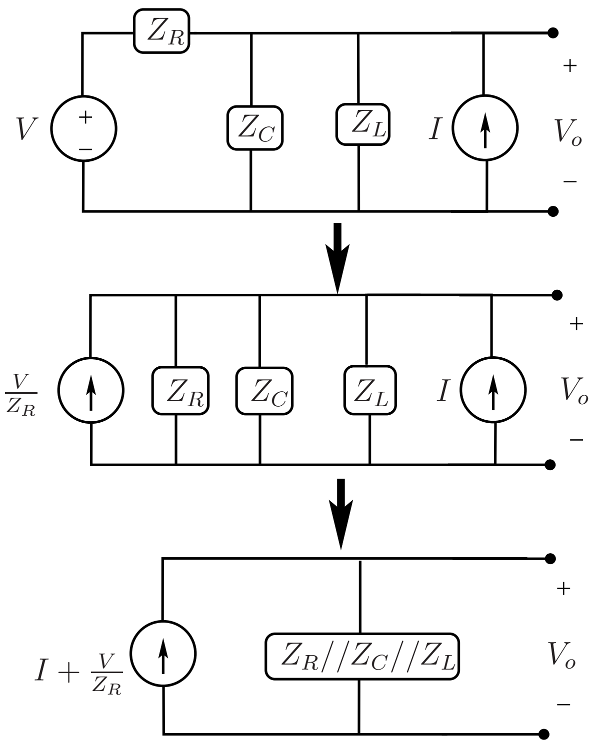
Problem 1.6 Recall the *Norton Equivalent* of an RLC circuit is a current source in parallel with a resistor and a capacitor or an inductor.

Find the value of C for which the Norton Equivalent is a current source in parallel with only a resistor (i.e. the Thevenin Impedance is purely real). What is $i_{sc}(t)$ and R_{th} is such a case?



where $v(t) = \cos(4t + \pi/3)$, $i(t) = \sin(4t + 5\pi/6)$, $R = 1\Omega$, *and* $L = 1/4H$. $C = ??$

Solution:



Since $v(t)$ and $i(t)$ are both sinusoidal with frequency 4, we can take the phasor transform with respect to $\omega = 4$.

Then by using a source transformation on the voltage source and Z_R, we have:
$I_{sc} = I + V/Z_R$ and

$$Z_{eff} = Z_R//Z_C//Z_L = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{1}{1 + j4C - j}.$$

If Z_{eff} is real, then $C = 1/4F$.

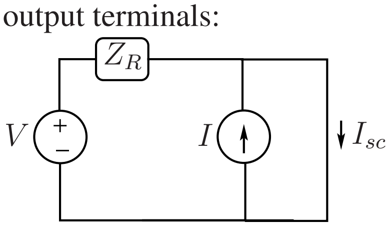
Finally,

$$i_{sc}(t) = i(t) + \frac{v(t)}{R} = 2\cos(4t + \pi/3)$$

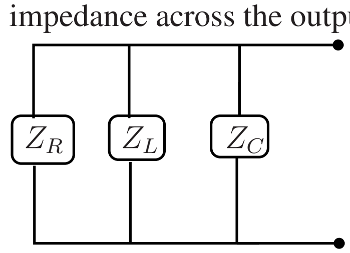
$$R_{th} = 1\Omega.$$

Alternatively:

We can solve for I_{sc} by shorting the output terminals:



We can solve for Z_{th} by setting $V = 0$ and $I = 0$ and solving for the effective impedance across the output terminals:



Problem 1.7 Suppose $H(\omega)$ is the transfer function of a linear system and $H(\omega) = 1$ when $|\omega| < \pi$ and $H(\omega) = 0$ otherwise. If the input to the system is

$$x(t) = \sum_{k=1}^{\infty} \frac{\cos\left(\frac{3\pi}{4}kt + \frac{\pi}{k}\right)}{k}$$

then what is the output $y(t)$?

Solution:

For each $k = 1, 2, \ldots$, let $x_k(t) = \frac{\cos\left(\frac{3\pi}{4}kt+\frac{\pi}{k}\right)}{k}$ and let $y_k(t)$ denote the output when $x_k(t)$ is the input. Then $x(t) = \sum_{k=1}^{\infty} x_k(t)$, so by the linearity of the system, $y(t) = \sum_{k=1}^{\infty} y_k(t)$.
For each $k = 1, 2, \ldots$, since $x_k(t)$ is sinusoidal with frequency $\frac{3\pi}{4}k$, we have

$$y_k(t) = \frac{\left|H\left(\frac{3\pi}{4}k\right)\right|}{k} \cos\left(\frac{3\pi}{4}kt + \frac{\pi}{k} + \angle H\left(\frac{3\pi}{4}k\right)\right) = \begin{cases} \cos(\frac{3\pi}{4}t + \pi) & k = 1 \\ 0 & \text{else} \end{cases}$$

Thus $y(t) = y_1(t) = -\cos(\frac{3\pi}{4}t)$.

Problem 1.8 Are the following steady-state input-output pairs consistent with the properties of RLC circuits (or more generally, LTI systems)?

(a) $\cos(2t) \rightarrow H(\omega) \rightarrow 99\sin(2t - e)$

(b) $\cos(4t) \rightarrow H(\omega) \rightarrow 1 + 4\cos(4t)$

(c) $4 \rightarrow H(\omega) \rightarrow -8$

(d) $4 \rightarrow H(\omega) \rightarrow 8j$

(e) $4 \rightarrow H(\omega) \rightarrow \cos(3t)$

(f) $\sin(\pi t) \rightarrow H(\omega) \rightarrow \cos(\pi t) + \sin(\pi t)$

(g) $\sin(\pi t) \rightarrow H(\omega) \rightarrow \sin^2(\pi t)$

(h) $0 \rightarrow H(\omega) \rightarrow 5$.

Solution:

(a) Yes. The output is an amplitude-scaled and phase-shifted version of the input.

(b) No. There is a new frequency component, $\omega = 0$ in the output.

(c) Yes. The output is an amplitude-scaled and phase-shifted version of the input.

(d) No. We cannot have imaginary values as output signals when the input is real.

(e) No. There is a new frequency component, $\omega = 3$ in the output.

(f) Yes. The output is an amplitude-scaled and phase-shifted version of the input, since $\cos(\pi t) + \sin(\pi t) = \sqrt{2}\cos(\pi t - \pi/4)$.

(g) No. There are two new frequency components, $\omega = 0$ and $\omega = 2\pi$, since $\sin^2(\pi t) = \frac{1}{2}(1 - \cos(2\pi t))$.

(h) No. We cannot have a non-zero output when the input is 0.

Problem 1.9 For each $k = 1, 2$, suppose $y_k(t)$ is the output when $x_k(t)$ is the input to an LTI system.

$$y_1(t) = \begin{cases} 1 & \text{if } 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases} \qquad y_2(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ -1 & \text{if } 1 \le t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine a possible input to the LTI system (in terms of $x_1(t)$ and $x_2(t)$) that could have yielded the following outputs:

(a) $z_1(t) = \begin{cases} -1 & \text{if } 2 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$ (c) $z_3(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$ (e) $z_5(t) = \begin{cases} 1 & \text{if } t \ge 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $z_2(t) = \begin{cases} 1 & \text{if } 1 \le t < 5 \\ 0 & \text{otherwise} \end{cases}$ (d) $z_4(t) = \begin{cases} 1 & \text{if } 3 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$ (f) $z_6(t) = 0$.

Solution:

(a) $z_1(t) = -y_1(t-2)$, so a possible input was $-x_1(t-2)$.

(b) $z_2(t) = y_1(t-1) + y_1(t-3)$, so a possible input was $x_1(t-1) + x_1(t-3)$.

(c) $z_3(t) = \frac{1}{2}(y_1(t) + y_2(t))$, so a possible input was $\frac{1}{2}(x_1(t) + x_2(t))$.

(d) $z_4(t) = z_3(t-3)\frac{1}{2}(y_1(t-3) + y_2(t-3))$, so a possible input was $\frac{1}{2}(x_1(t-3) + x_2(t-3))$.

(e) $z_5(t) = y_1(t) + y_1(t-2) + y_1(t-4) + \cdots$, so a possible input was $\sum_{k=0}^{\infty} x_k(t-2k)$.

(f) $z_6(t) = 0 = y_1(t) - y_1(t)$, so a possible input was $x_1(t) - x_1(t) = 0$.

Remark:

For an arbitrary LTI system, suppose $y(t)$ is the output when $x(t)$ is the input. Then when $0 = x(t) - x(t)$ is the input to the LTI system, by linearity, the output is $y(t) - y(t) = 0$.

Thus for any LTI system, if the input is 0, then the output must be 0.

However, the converse need not be true (i.e. if the output is 0, then the input is not necessarily 0). What are some simple examples of this? (e.g. High-pass filter with a DC input).

Problem 1.10 An LTI system with input $x(t)$ and output $y(t)$ is given by the differential equation

$$3\frac{d^4 y(t)}{dt^4} - 2\frac{d^3 x(t)}{dt} + y(t) = 2x(t) + \frac{d^2 y(t)}{dt}.$$

Find the steady-state output when $x(t) = 1 + \cos(t) + \cos(2t)$.

Solution:

Let $x_0(t) = 1$, $x_1(t) = \cos(t)$, and $x_2(t) = \cos(2t)$,

For each $k = 0, 1, 2$, let $y_k(t)$ be the output when $x_k(t)$ is the input.

Then $x(t) = x_0(t) + x_1(t) + x_2(t)$, so by the linearity of the system, $y(t) = y_0(t) + y_1(t) + y_2(t)$.

We can find the transfer function of the system by taking the phasor transformation of both sides of the differential equation:

$$3(j\omega)^4 Y - 2(j\omega)^3 X + Y = 2X + 2(j\omega)^2 Y$$
$$\Rightarrow H(\omega) = \frac{Y}{X} = \frac{2 - 2j\omega^3}{3\omega^4 + 2\omega^2 + 1}$$

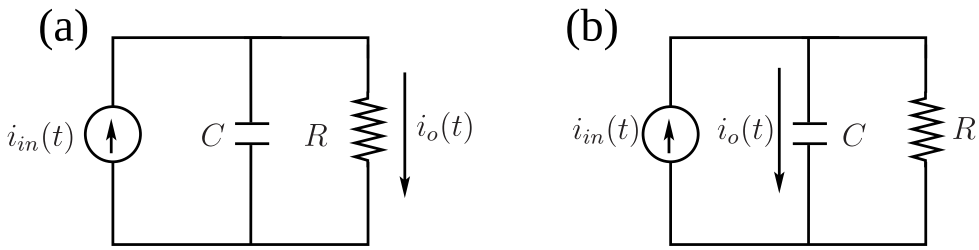
For each $k = 0, 1, 2$, let X_k and Y_k be the phasor transformations of $x_k(t)$ and $y_k(t)$ with frequency k, respectively. Then

$$Y_0 = H(0)X_0 = 2$$
$$\Rightarrow y_0(t) = 2$$
$$Y_1 = H(1)X_1 = \frac{\sqrt{2}}{3}e^{-j\pi/4}$$
$$\Rightarrow y_1(t) = \frac{\sqrt{2}}{3}\cos(t - \pi/4)$$
$$Y_2 = H(2)X_2 = \frac{2 - 16j}{57} = \frac{\sqrt{260}}{57}e^{-j\tan^{-1}(8)}$$
$$\Rightarrow y_2(t) = \frac{\sqrt{260}}{57}\cos(2t - \tan^{-1}(8)).$$

Finally, $y(t) = y_0(t) + y_1(t) + y_2(t) = 2 + \frac{\sqrt{2}}{3}\cos(t - \pi/4) + \frac{\sqrt{260}}{57}\cos(2t - \tan^{-1}(8))$.

Problem 1.11 Recall a *low-pass filter* only allows frequencies below some threshold, a *high-pass filter* only allows frequencies above some threshold, a *band-pass filter* only allows frequencies in some range, and a *band-reject filter* only allows frequencies outside of some range.

What type of filter are the following circuits? Justify your answer by finding the magnitude of the transfer function of each circuit.



Solution:

(a) We can use a current divider to get:

$$I_o = I_{in}\frac{1/Z_R}{1/Z_R + 1/Z_C}$$
$$\Rightarrow H(\omega) = \frac{I_o}{I_{in}} = \frac{1}{1 + Z_R/Z_C} = \frac{1}{1 + j\omega RC}$$
$$\Rightarrow |H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

When $\omega \approx 0$, $|H(\omega)| \approx 1$, and as $\omega \to \infty$, $|H(\omega)| \approx 0$. Thus this is a low-pass filter.

(b) We can again use a current divider to get:

$$\begin{aligned} I_o &= I_{in} \frac{1/Z_C}{1/Z_R + 1/Z_C} \\ \Rightarrow H(\omega) &= \frac{I_o}{I_{in}} = \frac{1}{1 + Z_C/Z_R} = \frac{1}{1 + \frac{1}{j\omega RC}} \\ \Rightarrow |H(\omega)| &= \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} \end{aligned}$$

When $\omega \approx 0$, $|H(\omega)| \approx 0$, and as $\omega \to \infty$, $|H(\omega)| \approx 1$. Thus this is a high-pass filter.

Problem 1.12 For each of the circuits in the previous problem, let $RC = 100$.
If the circuit is a low-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega > \omega_c$.
If the circuit is a high-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega < \omega_c$.
If $RC = \beta$ for some $\beta > 0$, what is ω_c in terms of β?

Solution:

(a) The magnitude of the low-pass filter is given by

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Suppose $|H(\omega)| < \frac{1}{100}$. Then we have

$$\begin{aligned} \sqrt{1 + (\omega RC)^2} &> 100 \\ \Rightarrow \omega^2 &> \frac{100^2 - 1}{(RC)^2} \approx \left(\frac{100}{RC}\right)^2 \\ \Rightarrow \omega &> \frac{100}{\beta}. \end{aligned}$$

(a) The magnitude of the high-pass filter is given by

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}$$

Suppose $|H(\omega)| < \frac{1}{100}$. Then we have

$$\begin{aligned} \sqrt{1 + \left(\frac{1}{\omega RC}\right)^2} &> 100 \\ \Rightarrow \omega^2 &< \frac{1}{(RC)^2(100^2 - 1)} \approx \left(\frac{1}{100RC}\right)^2 \\ \Rightarrow \omega &< \frac{1}{100\beta}. \end{aligned}$$

MATLAB Problem 1 Download and load the file "sum.mat" into MATLAB by placing it in your MATLAB directory and running "load sum;" The entries of the array sum are given as follows:

$$\begin{aligned} sum[1] &= z[1] \\ sum[2] &= z[1] + z[2] \\ sum[3] &= z[1] + z[2] + z[3] \\ &\vdots \\ sum[N-1] &= z[1] + z[2] + \cdots + z[N-1] \\ sum[N] &= z[1] + z[2] + \cdots + z[N] \end{aligned}$$

where z is an array containing an audio message. In other words, the kth entry of sum is the sum of the first k entries of z. Your goal is to use MATLAB to recover the audio message z from the array sum and play it within MATLAB, using "sound(z, F_s);" where $F_s = 11025$. The array z has the same length as the array sum.

- What is the (well-known) audio message? Include the code you used to decipher z.
- In MATLAB, let $N =$ length(sum); and define the array $t = (1 : N)/F_s$; Include the output of: "plot(t, z);" and label:
  - the x-axis as "Time (seconds)"
  - the y-axis as "Amplitude"
  - the title as whatever you deem appropriate.
- When running "sound(z, F_s);" what happens if you set $F_s = 5000$, instead of 11025? What if $F_s = 20000$?

Solutions

Since $\text{sum}[n] = \sum_{k=1}^{n} z[n]$, we can recover $z[n]$ as follows:

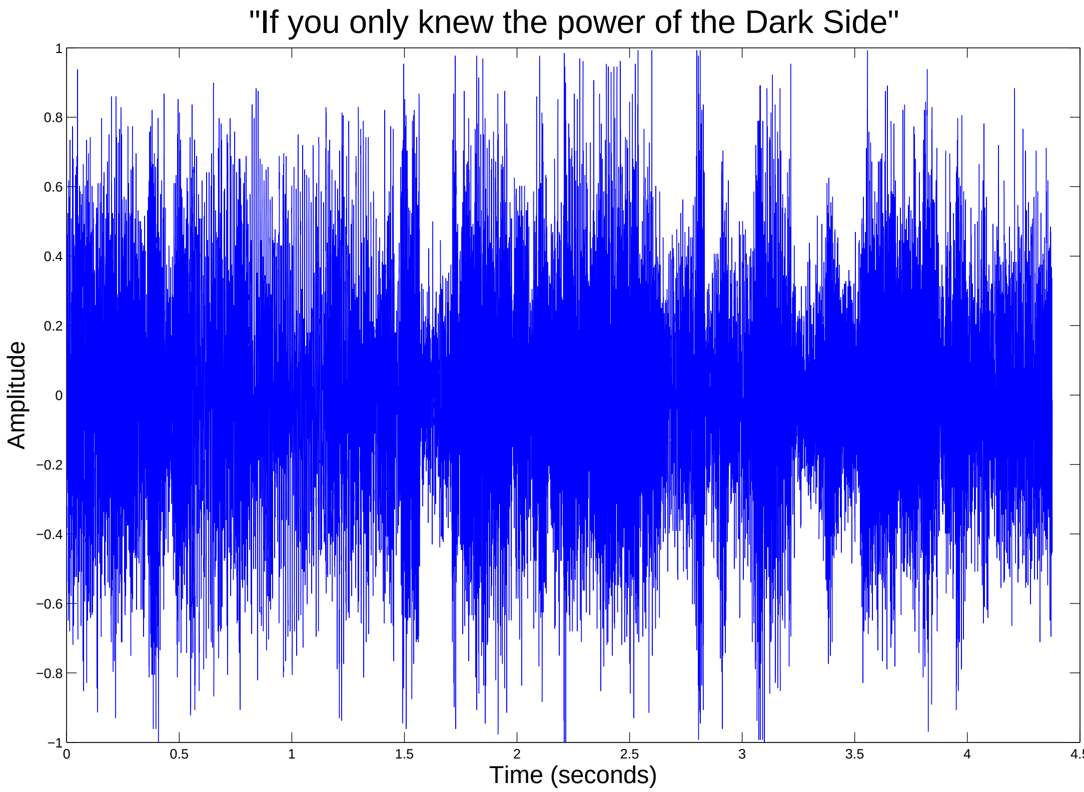
$$z[1] = \text{sum}[1] \;\text{ and }\; z[n] = \text{sum}[n] - \text{sum}[n-1] \text{ for } n \geq 2.$$

See ECE45_MATLAB1_1.m for MATLAB script.

"If only you knew the power of the Dark Side."
- Darth Vader (Star Wars: Episode V - The Empire Strikes Back)

When $Fs = 5000$, MATLAB is assuming the signal was sampled at 5000 Hz (when in fact, it was sampled at 11025 Hz), so its playback speed is roughly halved. Similarly, when $Fs = 20000$, the playback speed is roughly doubled.

Fs is the frequency used to sample to .wav file I used to generate this signal. When you play back an audio file in MATLAB, it needs to know the frequency at which the signal was sampled. We will talk more about sampling later in the course.



MATLAB Problem 2 Let N be a positive integer, and define the length-4000 arrays x_1, x_2, and x_3 as follows:

$$x_1[k] = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{N} \frac{\sin\left(\frac{\pi n}{500}k\right)}{n}$$

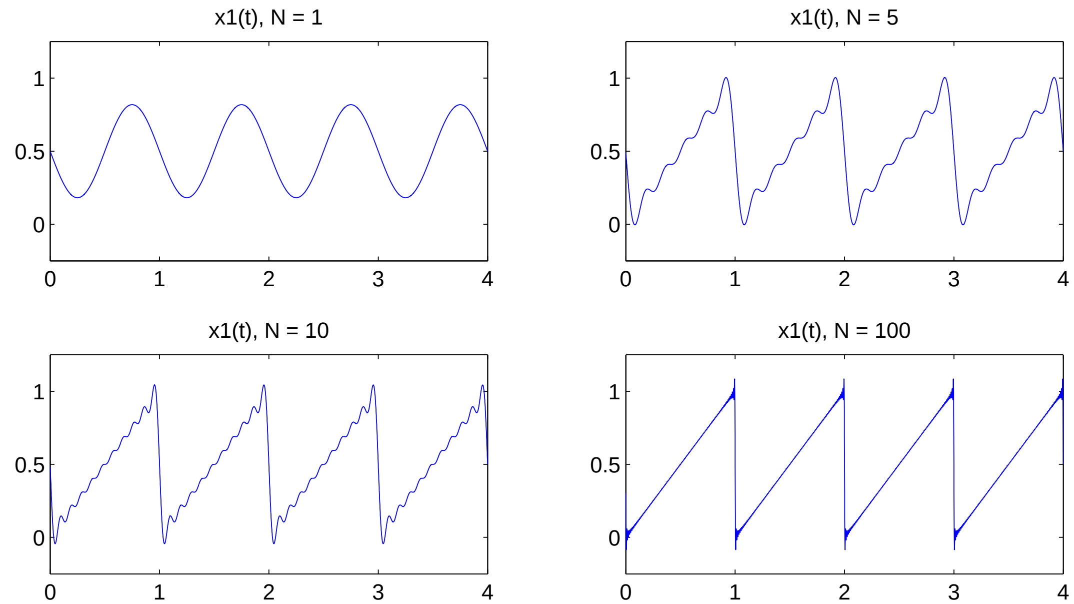
$$x_2[k] = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{N} \frac{\cos\left(\frac{\pi n}{500}k\right)}{4n^2-1}$$

$$x_3[k] = \frac{1}{2} + \frac{2}{\pi}\sum_{\substack{n=1\\ n \text{ odd}}}^{2N-1} \frac{\sin\left(\frac{\pi n}{500}k\right)}{n}$$

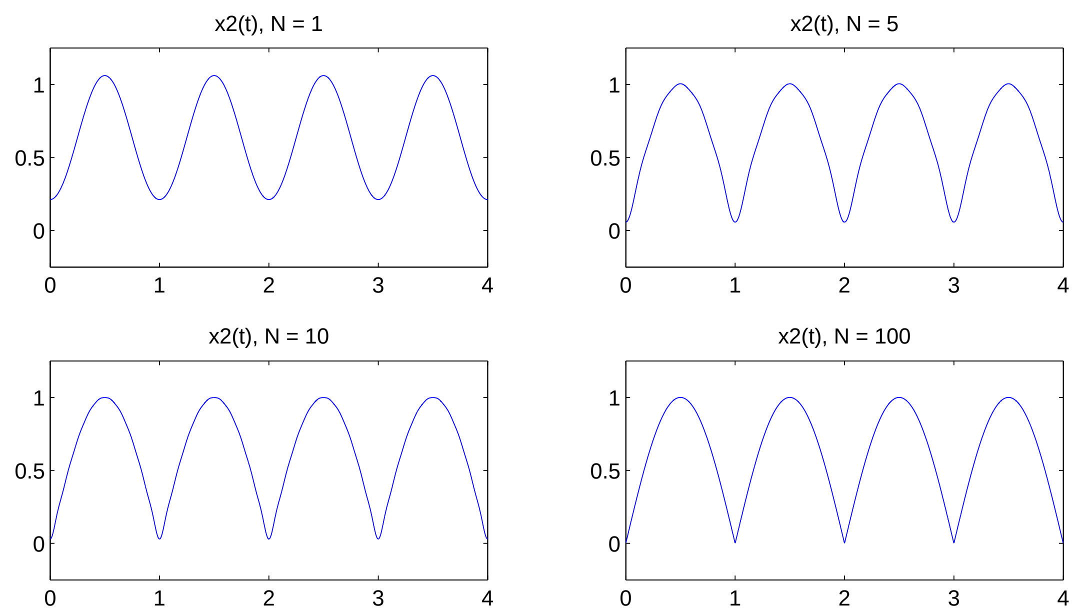
where $x_i[k]$ refers to the kth entry of the array x_i and k ranges from 1 to 4000.

- Create an array $t = (1 : 4000)/1000$; and include the output of "plot(t, x_1);" "plot(t, x_2);" and "plot(t, x_3);" when $N = 1, 5, 10$, and 100. Be sure to label your plots.
- What do you notice about these functions as N grows large? In each case, what function is being approximated?

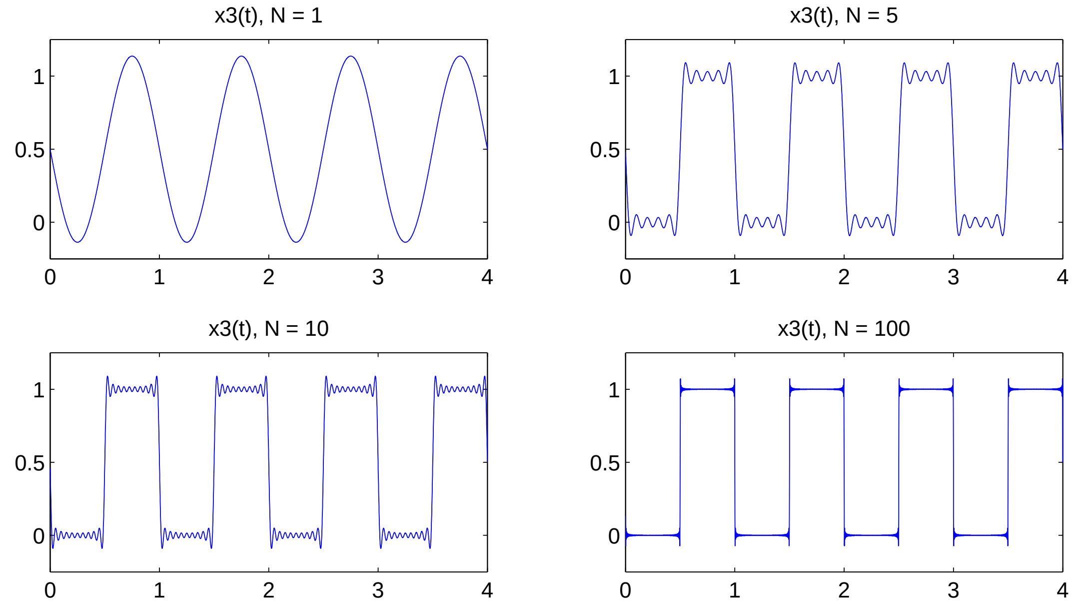
Solutions



As N grows large, $x_1(t)$ converges to a Sawtooth wave that repeats every 1 time unit.



As N grows large, $x_2(t)$ converges to $|\sin(2\pi t)|$.



As N grows large, $x_3(t)$ converges to a square wave that repeats every 1 time unit.

In each case, as N grows, we are adding sinusoids with higher and higher frequencies to the signal $x_i(t)$, which yields a better approximation of the desired signal.