UC San Diego Spring 2018

ECE 45 Homework 1

Problem 1.1 Find the magnitude and phase of the following complex number

$$\frac{(1-j)\,(2e^{-j\pi/3})\,(\sin(1))\,(j^2)}{(2+2j)\,(-j\cos(1))\,(e^{j\pi})}$$

without using a calculator. Simplify as much as possible.

Problem 1.2 Let $X = 2 + j + 2e^{-j2\pi/3} - e^{j\pi/2}$.

(a) Find the real portion of X.

(b) Find the phase of the complex conjugate of X.

(c) Let $Y = -2 + 2j$. Plot X/Y on the complex plane (real and imaginary axes).

Problem 1.3 Let $f(x) = \begin{cases} 1 & \text{if } -1 \le x \le 0 \\ -1 & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $f(2x+1)$.

(b) Plot the magnitude of $e^{jx}\,f(x)$.

(c) If the *energy* of a signal $g(x)$ is defined to be

$$E[g(x)] = \int_{-\infty}^{\infty} |g(x)|^2\,dx$$

how does the energy of $g(x)$ compare to the energy of $e^{jx}\,g(x)$?

Problem 1.4 Represent the following sinusoidal functions as phasors.

(a) $f_1(t) = 3\cos(4t) - 4\sin(4t)$

(b) $f_2(t) = 2(\cos(\omega t) + \cos(\omega t + \pi/4))$

(c) $f_3(t) = \cos^2(t) - \sin^2(t)$

Please report any typos/errors to j2connelly@ucsd.edu

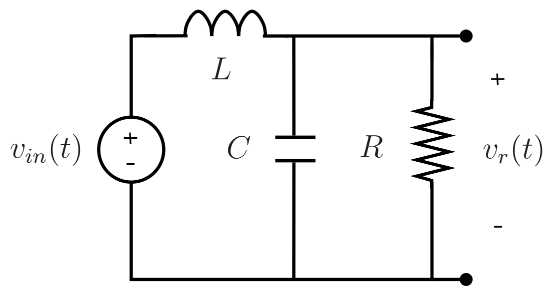
Problem 1.5 Find the voltage $v_r(t)$ in the circuit below, when

(a) $v_{in}(t) = 1/3$

(b) $v_{in}(t) = \sin(t)$

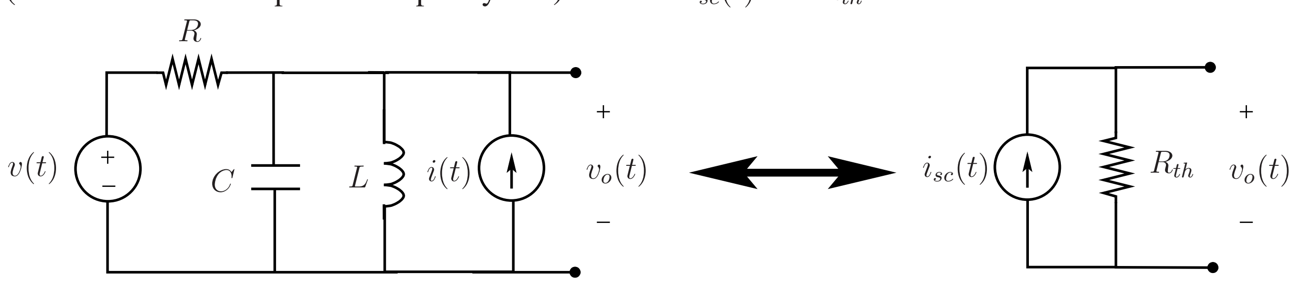
(c) $v_{in}(t) = 1 + 2\sin(t - \pi)$.

where $R = 1\Omega$, $C = 1/2F$, $L = 2H$.



Problem 1.6 Recall the *Norton Equivalent* of an RLC circuit is a current source in parallel with a resistor and a capacitor or an inductor.

Find the value of C for which the Norton Equivalent is a current source in parallel with only a resistor (i.e. the Thevenin Impedance is purely real). What is $i_{sc}(t)$ and R_{th} is such a case?



where $v(t) = \cos(4t + \pi/3)$, $i(t) = \sin(4t + 5\pi/6)$, $R = 1\Omega$, and $L = 1/4H$.

Problem 1.7 Suppose $H(\omega)$ is the transfer function of a linear system and $H(\omega) = 1$ when $|\omega| < \pi$ and $H(\omega) = 0$ otherwise. If the input to the system is

$$x(t) = \sum_{k=1}^{\infty} \frac{\cos\left(\frac{3\pi}{4}kt + \frac{\pi}{k}\right)}{k}$$

then what is the output $y(t)$?

Problem 1.8 Are the following steady-state input-output pairs consistent with the properties of RLC circuits (or more generally, LTI systems)?

(a) $\cos(2t) \to H(\omega) \to 99\sin(2t - e)$

(b) $\cos(4t) \to H(\omega) \to 1 + 4\cos(4t)$

(c) $4 \to H(\omega) \to -8$

(d) $4 \to H(\omega) \to 8j$

(e) $4 \to H(\omega) \to \cos(3t)$

(f) $\sin(\pi t) \to H(\omega) \to \cos(\pi t) + \sin(\pi t)$

(g) $\sin(\pi t) \to H(\omega) \to \sin^2(\pi t)$

(h) $0 \to H(\omega) \to 5$.

Problem 1.9 For each $k = 1, 2$, suppose $y_k(t)$ is the output when $x_k(t)$ is the input to an LTI system.

$$y_1(t) = \begin{cases} 1 & \text{if } 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases} \qquad y_2(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ -1 & \text{if } 1 \le t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine a possible input to the LTI system (in terms of $x_1(t)$ and $x_2(t)$) that could have yielded the following outputs:

(a) $z_1(t) = \begin{cases} -1 & \text{if } 2 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$ (c) $z_3(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$ (e) $z_5(t) = \begin{cases} 1 & \text{if } t \ge 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $z_2(t) = \begin{cases} 1 & \text{if } 1 \le t < 5 \\ 0 & \text{otherwise} \end{cases}$ (d) $z_4(t) = \begin{cases} 1 & \text{if } 3 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$ (f) $z_6(t) = 0$.

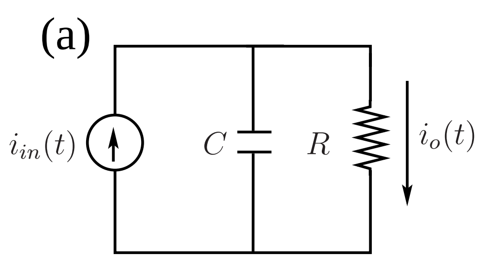
Problem 1.10 An LTI system with input $x(t)$ and output $y(t)$ is given by the differential equation

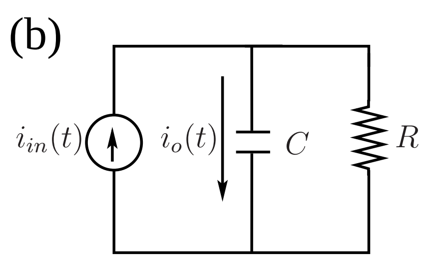
$$3\frac{d^4y(t)}{dt^4} - 2\frac{d^3x(t)}{dt} + y(t) = 2x(t) + \frac{d^2y(t)}{dt}.$$

Find the steady-state output when $x(t) = 1 + \cos(t) + \cos(2t)$.

Problem 1.11 Recall a *low-pass filter* only allows frequencies below some threshold, a *high-pass filter* only allows frequencies above some threshold, a *band-pass filter* only allows frequencies in some range, and a *band-reject filter* only allows frequencies outside of some range.

What type of filter are the following circuits? Justify your answer by finding the magnitude of the transfer function of each circuit.





Problem 1.12 For each of the circuits in the previous problem, let $RC = 100$.

If the circuit is a low-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega > \omega_c$.

If the circuit is a high-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega < \omega_c$.

If $RC = \beta$ for some $\beta > 0$, what is ω_c in terms of β?

MATLAB Problem 0 Download "MATLAB_Intro.html" for an introduction on how to import and export data, play sound files, and plot in MATLAB. These will be important concepts you will use throughout the quarter. You can use "test.wav" as an example audio file.

MATLAB Problem 1 Download and load the file "sum.mat" into MATLAB by placing it in your MATLAB directory and running "load sum;" The entries of the array sum are given as follows:

$$\begin{aligned} sum[1] &= z[1] \\ sum[2] &= z[1] + z[2] \\ sum[3] &= z[1] + z[2] + z[3] \\ &\vdots \\ sum[N-1] &= z[1] + z[2] + \cdots + z[N-1] \\ sum[N] &= z[1] + z[2] + \cdots + z[N] \end{aligned}$$

where z is an array containing an audio message. In other words, the kth entry of sum is the sum of the first k entries of z. Your goal is to use MATLAB to recover the audio message z from the array sum and play it within MATLAB, using "sound(z, F_s);" where $F_s = 11025$. The array z has the same length as the array sum.

- What is the (well-known) audio message? Include the code you used to decipher z.
- In MATLAB, let N = length(sum); and define the array $t = (1 : N)/F_s$; Include the output of: "plot(t, z);" and label:
  - the x-axis as "Time (seconds)"
  - the y-axis as "Amplitude"
  - the title as whatever you deem appropriate.
- When running "sound(z, F_s);" what happens if you set $F_s = 5000$, instead of 11025? What if $F_s = 20000$?

MATLAB Problem 2 Let N be a positive integer, and define the length-4000 arrays x_1, x_2, and x_3 as follows:

$$\begin{aligned} x_1[k] &= \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{N} \frac{\sin\left(\frac{\pi n}{500}k\right)}{n} \\ x_2[k] &= \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{\cos\left(\frac{\pi n}{500}k\right)}{4n^2-1} \\ x_3[k] &= \frac{1}{2} + \frac{2}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{2N-1} \frac{\sin\left(\frac{\pi n}{500}k\right)}{n} \end{aligned}$$

where $x_i[k]$ refers to the kth entry of the array x_i and k ranges from 1 to 4000.

- Create an array $t = (1 : 4000)/1000$; and include the output of "plot(t, x_1);" "plot(t, x_2);" and "plot(t, x_3);" when $N = 1, 5, 10$, and 100. Be sure to label your plots.
- What do you notice about these functions as N grows large? In each case, what function is being approximated?

Fs is the frequency used to sample to .wav file I used to generate this signal. When you play back an audio file in MATLAB, it needs to know the frequency at which the signal was sampled. We will talk more about sampling later in the course.