ECE 45 Homework 3 Solutions

Problem 3.1 Calculate the Fourier transform of the function

$$\Delta(t) = \begin{cases} 1 - 2|t| & |t| \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

Use the statement of **Problem 3.2** to verify your answer. **Note:** the function $\Delta(t)$ is sometimes called the *unit triangle function*, as it a triangular pulse with height 1, width 1, and is centered at 0.

Hint: Recall the trig identity $1 - \cos(2x) = 2\sin^2(x)$. Also, the time-reversal property can be helpful.

Solution:

$$\text{Let } z(t) = \begin{cases} 1 - 2t & 0 \leq t \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

Then $\Delta(t) = z(t) + z(-t)$, so by the linearity and time reversal properties of the Fourier Transform, $F(\omega) = Z(\omega) + Z(-\omega)$.

$$\begin{aligned} Z(\omega) &= \int_{-\infty}^{\infty} z(t) e^{-j\omega t}\, dt \\ &= \int_0^{1/2} (1 - 2t) e^{-j\omega t}\, dt \\ &= \frac{j\omega\,(2t - 1) + 2}{(j\omega)^2} e^{-j\omega t}\Big|_{t=0}^{1/2} \\ &= \frac{2 - j\omega - 2e^{-j\omega/2}}{\omega^2} \end{aligned}$$

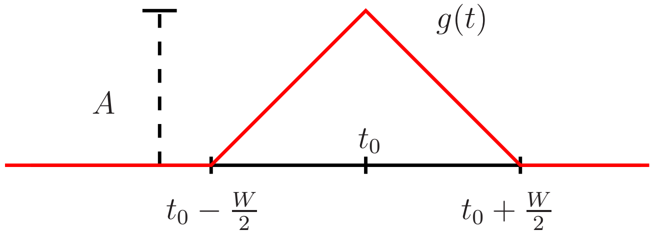
Therefore

$$\begin{aligned} D(\omega) &= \frac{2 - j\omega - 2e^{-j\omega/2}}{\omega^2} + \frac{2 + j\omega - 2e^{j\omega/2}}{\omega^2} \\ &= \frac{4 - 2\left(e^{j\omega/2} + e^{-j\omega/2}\right)}{\omega^2} \\ &= \frac{4 - 4\,\cos(\omega/2)}{\omega^2} \\ &= \frac{8\sin^2(\omega/4)}{\omega^2} = \frac{\text{sinc}^2\,(\omega/4)}{2} \end{aligned}$$

where $\text{sinc}\,(x) = \frac{\sin x}{x}$. This corresponds to the function in Problem 3.3 with $A = W = 1$ and $t_0 = 0$.

Please report any typos/errors to j2connelly@ucsd.edu

Problem 3.2 Let A, W, and t_0 be real numbers such that $A, W > 0$, and suppose that $g(t)$ is given by



Show the Fourier transform of $g(t)$ is equal to

$$\frac{AW}{2}\,\mathrm{sinc}^2\left(W\omega/4\right)\, e^{-j\omega t_0}$$

using the results of Problem 3.1 and the properties of the Fourier transform.

Hint: You do NOT have to re-integrate, this should only take a few lines.

Solution:

$g(t)$ is a triangular pulse of height A, width W, and is centered at t_0. $\Delta(t)$, from Problem 3.1, is a triangular pulse of height 1, width 2, and is centered at 0. Thus $g(t)$ is an amplitude-scaled, time-scaled, time-shifted version of $\Delta(t)$. In particular, $g(t) = A\Delta\left(\frac{t-t_0}{W}\right)$.

By the amplitude scaling, time scaling, and time shift properties:

$$\begin{aligned} G(\omega) &= A\,\frac{1}{\left|\frac{1}{W}\right|}\,D\left(\frac{\omega}{\frac{1}{W}}\right)\,e^{-j\omega t_0} \\ &= AW\,D(W\omega)\,e^{-j\omega t_0} \\ &= \frac{AW}{2}\,\mathrm{sinc}^2\left(W\omega/4\right)\,e^{-j\omega t_0} \end{aligned}$$

Problem 3.3 Find the Fourier transform of the function

$$x(t) = \begin{cases} 1 & \text{if } 1 \le |t| \le 3 \\ -1 & \text{if } |t| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Hint: Recall rectangle functions to reduce amount of integration.

Solution:

Recall the **unit rectangle function**

$$\text{rect}\,(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

We denote the Fourier transform of $\text{rect}\,(t)$ by $R(\omega)$. Then

$$\begin{aligned} R(\omega) &= \int_{-\infty}^{\infty} \text{rect}\,(t)\, e^{-j\omega t}\, dt \\ &= \int_{-1/2}^{1/2} e^{-j\omega t}\, dt \\ &= \frac{1}{-j\omega}(e^{-j\omega/2} - e^{j\omega/2}) \\ &= \frac{2}{\omega}\sin(\omega/2) = \text{sinc}\,(\omega/2)\,. \end{aligned}$$

We have $x(t) = \text{rect}\left(\frac{t}{6}\right) - 2\,\text{rect}\left(\frac{t}{2}\right)$, so by the time-scaling property, we have

$$X(\omega) = 6R(6\omega) - 4R(2\omega) = 6\,\text{sinc}\,(3\omega) - 4\,\text{sinc}\,(\omega)\,.$$

Alternatively, $x(t) = \text{rect}\left(\frac{t+2}{2}\right) - \text{rect}\left(\frac{t}{2}\right) + \text{rect}\left(\frac{t-2}{2}\right)$, so by the time-scaling and time-shift properties, we have

$$\begin{aligned} X(\omega) &= 2e^{j2\omega}R(2\omega) - 4R(2\omega) + 2e^{-j2\omega}R(2\omega) \\ &= 2R(2\omega)\,(e^{j2\omega} + e^{-j2\omega} - 2) \\ &= 4R(2\omega)\,(\cos(2\omega) - 1) = 4\,\text{sinc}\,(\omega/2)\,(\cos(2\omega) - 1). \end{aligned}$$

It can be shown with trig manipulations that these two functions are equal.

Problem 3.4 Find the inverse Fourier transform of the function

$$F(\omega) = \frac{12 + 7j\omega - \omega^2}{(\omega^2 - 2j\omega - 1)(-\omega^2 + j\omega - 6)}$$

Hint: Use Partial fractions.

Solution:

By factoring each of the quadratic polynomials, we have

$$\begin{aligned} F(\omega) &= \frac{(3 + j\omega)(4 + j\omega)}{(1 + j\omega)^2(2 - j\omega)(3 + j\omega)} \\ &= \frac{(4 + j\omega)}{(1 + j\omega)^2(2 - j\omega)} = \frac{A}{(1 + j\omega)^2} + \frac{B}{1 + j\omega} + \frac{C}{2 - j\omega} \end{aligned}$$

where $(4+j\omega) = A(2-j\omega) + B(1+j\omega)(2-j\omega) + C(1+j\omega)^2$. Therefore

$$\begin{aligned}4 &= 2A + 2B + C\\ 1 &= -A + B + 2C\\ 0 &= B - C\end{aligned}$$

which yields, $A = 1$ and $B = C = \frac{2}{3}$.

From Discussion Notes 6, for any $a > 0$, we have

$$\begin{aligned}\mathcal{F}(e^{-at}u(t)) &= \frac{1}{a+j\omega}\\ \mathcal{F}(e^{at}u(-t)) &= \frac{1}{a-j\omega}\\ \mathcal{F}(te^{-at}u(t)) &= \frac{1}{(a+j\omega)^2}\end{aligned}$$

where $u(t)$ is the **unit step function.**

Thus by the linearity of the Fourier transform,

$$f(t) = te^{-t}u(t) + \frac{2}{3}e^{-t}u(t) + \frac{2}{3}e^{2t}u(-t) = \begin{cases}\left(t+\frac{2}{3}\right)e^{-t} & t \geq 0\\ \frac{2}{3}\,e^{2t} & t < 0.\end{cases}$$

Problem 3.5 Suppose a function $f(t)$ has Fourier transform

$$F(\omega) = 2\pi j\omega e^{-|\omega|}.$$

Is $f(t)$ purely real? Is $f(t)$ purely imaginary? Is $f(t)$ even? Is $f(t)$ odd? What is $f(0)$? Calculate $f(t)$ and verify these properties.

Hint: For finding the inverse Fourier transform, use this property: Let G(w) = 2πe^(−|ω|) and F(ω) = jωG(ω) and use the derivative property.

Solution:

Recall the Fourier transform is a one-to-one mapping, so $y(t) = z(t)$ if and only if $Y(\omega) = Z(\omega)$. For any function $x(t)$,

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(W)e^{-jWt}\,dW = \frac{1}{2\pi}\int_{\infty}^{-\infty} X^*(-\omega)e^{j\omega t}\,(-d\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(-\omega)e^{j\omega t}\,d\omega = \mathcal{F}^{-1}(X^*(-\omega))$$

Therefore, the Fourier transform of $x^*(t)$ is $X^*(-\omega)$.

If $x(t)$ is real, then $x(t) = x^*(t)$, so $X(\omega) = X^*(-\omega)$.

If $x(t)$ is imaginary, then $x(t) = -x^*(t)$, so $X(\omega) = -X^*(-\omega)$.

$$x(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(W)e^{-jWt}\,dW = \frac{1}{2\pi}\int_{\infty}^{-\infty} X(-\omega)e^{j\omega t}\,(-d\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(-\omega)e^{j\omega t}\,d\omega = \mathcal{F}^{-1}(X(-\omega))$$

Therefore, the Fourier transform of $x(-t)$ is $X(-\omega)$.

If a function $x(t)$ is even, then $x(t) = x(-t)$, so $X(\omega) = X(-\omega)$.

If a function $x(t)$ is odd, then $x(t) = -x(t)$, so $X(\omega) = -X(-\omega)$.

For the function $f(t)$, we have

$$F^*(-\omega) = \left(-2\pi j\omega e^{-|-\omega|}\right)^* = 2\pi j\omega e^{-|\omega|} = F(\omega)$$
$$F(-\omega) = -2\pi j\omega e^{-|-\omega|} = -2\pi j\omega e^{-|\omega|} = -F(\omega)$$

Therefore, $f(t)$ is real and odd. For any odd function, $f(0) = -f(-0) = -f(0) = 0$.

Let $G(\omega) = 2\pi e^{-|\omega|}$. Then $F(\omega) = j\omega G(\omega)$, so by the time-derivative property $f(t) = \frac{d}{dt}g(t)$.

$$\begin{aligned}
g(t) &= \frac{1}{2\pi}\int_{-\infty]}^{\infty} G(\omega)e^{j\omega t}\,d\omega = \int_{-\infty}^{\infty} e^{|\omega|}e^{j\omega t}\,d\omega \\
&= \int_{-\infty}^{0} e^{\omega}e^{j\omega t}\,d\omega + \int_{0}^{\infty} e^{-\omega}e^{j\omega t}\,d\omega \\
&= \int_{-\infty}^{0} e^{\omega(1+jt)}\,d\omega + \int_{0}^{\infty} e^{\omega(jt-1)}\,d\omega \\
&= \frac{1}{1+jt} + \frac{1}{1-jt} = \frac{(1-jt)+(1+jt)}{(1+jt)(1-jt)} = \frac{2}{1+t^2}
\end{aligned}$$

$$\text{Hence } f(t) = \frac{d}{dt}\frac{1}{1+t^2} = \frac{-2t}{(1+t^2)^2}$$

We also have $f(0) = 0$, $f(t) = -f(-t)$, and $f(t)$ is real, thus verifying our claims.

Problem 3.6 Suppose $x(t)$ is the input to an LTI system with transfer function $H(\omega)$, and $y(t)$ is the output of this system, where

$$x(t) = e^{-|t|}\cos(At) \text{ and } H(\omega) = 1 + e^{-j\omega} + e^{-3j\omega}.$$

Find a real number $A > 0$ such that $y(0) = 1$. Is your answer unique?

Solution:

We have

$$Y(\omega) = X(\omega)H(\omega) = X(\omega) + e^{-j\omega}X(\omega) + e^{-j3\omega}X(\omega).$$

Therefore by the linearity and time-shift properties of the Fourier transform,

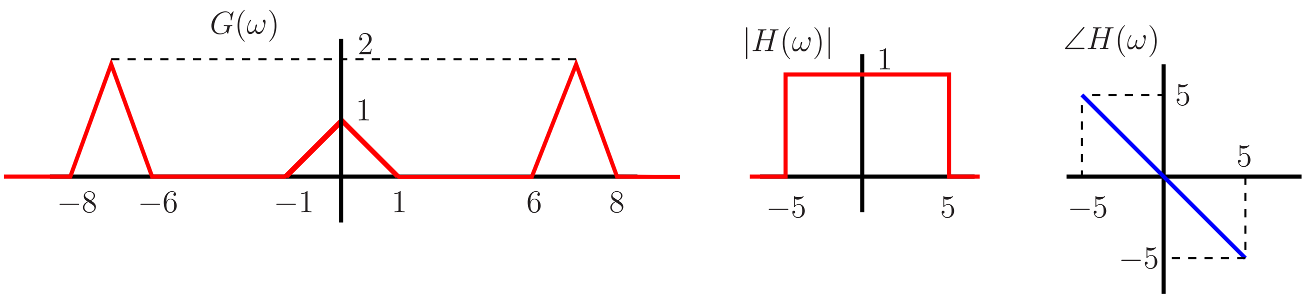
$$y(t) = x(t) + x(t-1) + x(t-3).$$

Then $y(0) = x(0) + x(-1) + x(-3)$.

$$\begin{aligned} x(0) &= e^{0}\ \cos(0) = 1 \\ x(-1) &= e^{-1}\ \cos(-A) \\ x(-3) &= e^{-3}\ \cos(-3A). \end{aligned}$$

If $A = \pi(n + 1/2)$, for any integer n, we have $x(-1) = x(-3) = 0$, so $y(0) = x(0) = 1$.

Problem 3.7 Suppose $g(t)$ is the input to an LTI system with transfer function $H(\omega)$, and $G(\omega)$ is the Fourier transform of $g(t)$. Find the output of the system $y(t)$.



Solution:

$$\text{Let } X(\omega) = \begin{cases} 1 - |\omega| & |\omega| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

In the interval $[-5, 5]$, $|H(\omega)| = 1$ and $\angle H(\omega) = -\omega$, and $H(\omega) = 0$ otherwise.
So, $Y(\omega) = H(\omega)G(\omega) = X(\omega)e^{-j\omega}$, which, by the time-shifting property of the Fourier transform, implies $y(t) = x(t-1)$.

$$\begin{aligned} x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega &= \frac{1}{2\pi}\int_{-1}^{0}(1+\omega)e^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{0}^{1}(1-\omega)e^{j\omega t}\,d\omega \\ &= \frac{jt - e^{jt} + 1}{2\pi t^2} - \frac{jt - e^{-jt} - 1}{2\pi t^2} \\ &= \frac{2 - 2\cos(t)}{2\pi t^2} \\ &= \frac{4\sin^2(t/2)}{2\pi t^2} \\ &= \frac{\text{sinc}^2\,(t/2)}{2\pi} \end{aligned}$$

Thus we have

$$y(t) = \frac{\text{sinc}^2\left(\frac{t-1}{2}\right)}{2\pi}$$

Alternatively, we can use the *Duality Property* and our results from Problem 3.3.

Duality Property:

If the Fourier transform of $f(t)$ is $F(\omega)$, then the Fourier transform of $F(t)$ is $2\pi f(-\omega)$.

Let $f(t)$ be a triangular pulse of height $\frac{1}{2\pi}$, width 2, centered at 0. Then $F(\omega) = \frac{1}{2\pi}\text{sinc}^2\,(\omega/2)$.

We have $X(\omega) = 2\pi f(\omega)$, and since f is an even function, $X(\omega) = 2\pi f(-\omega)$.

Therefore, by the Duality Property, the Fourier transform of $F(t)$ is $X(\omega)$, so $x(t) = F(t) = \frac{1}{2\pi}\text{sinc}^2\,(t/2)$.

Problem 3.8 Recall the *unit step function* $u(t)$ is given by $u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$

Suppose we have a system for which the output $y(t)$ is

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

when the input is $x(t)$. Find $y(t)$ and its Fourier transform $Y(\omega)$ when the input is

$$x(t) = u(t+1) - 2u(t-1) + u(t-3).$$

Solution:

$$\begin{aligned} y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau &= \begin{cases} \int_{-1}^{t} 1\, d\tau & -1 \leq t < 1 \\ \int_{-1}^{1} 1\, d\tau - \int_{1}^{t} 1\, d\tau & 1 \leq t < 3 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 1+t & -1 \leq t < 1 \\ 3-t & 1 \leq t < 3 \\ 0 & \text{otherwise} \end{cases} \\ &= 2\,\Delta\left(\frac{t-1}{4}\right) \end{aligned}$$

By Problem 3.3 with $A = 2$, $W = 4$, and $t_0 = 1$, we have

$$Y(\omega) = 4\,\text{sinc}^2\,(\omega)\; e^{-j\omega}$$

Problem 3.9 An LTI system has impulse response $h(t) = e^{-3t}u(t)$. What was the input $x(t)$, when the output is $e^{-3t}u(t) - e^{-4t}u(t)$?

Solution:

For all $a > 0$, we have

$$\mathcal{F}(e^{-at}u(t)) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-t(a+j\omega)}\, dt = \frac{1}{a+j\omega}$$

Therefore $H(\omega) = \dfrac{1}{3+j\omega}$ and $Y(\omega) = \dfrac{1}{3+j\omega} - \dfrac{1}{4+j\omega} = \dfrac{1}{(3+j\omega)}\dfrac{1}{(4+j\omega)}$.

We also have

$$Y(\omega) = X(\omega)H(\omega) = X(\omega)\,\frac{1}{3+j\omega}$$

Thus $X(\omega) = \dfrac{1}{4+j\omega}$, so $x(t) = u(t)e^{-4t}$.

Problem 3.10 Let $x(t) = u(t)e^{-3t}$. Find $y(t)$ when

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} - 2y(t) = x(t).$$

Hint: Use Partial Fractions.

Solution:

For all $a > 0$, we have $\mathcal{F}(e^{-at}u(t)) = \frac{1}{a+j\omega}$ and $\mathcal{F}(e^{at}u(-t)) = \frac{1}{a-j\omega}$

By taking the Fourier transform of both sides of the differential equation, we have

$$(j\omega)^2\, Y(\omega) + j\omega Y(\omega) - 2Y(\omega) = X(\omega) = \frac{1}{3 + j\omega}$$

Therefore

$$Y(\omega) = \frac{X(\omega)}{(j\omega)^2 + j\omega - 2} = \frac{1}{(3 + j\omega)(j\omega - 1)(j\omega + 2)} = \frac{A}{3 + j\omega} + \frac{B}{j\omega - 1} + \frac{C}{2 + j\omega}$$

Solving for A, B, and C gives us

$$\begin{aligned} A(j\omega - 1)(2 + j\omega) + B(3 + j\omega)(2 + j\omega) + C(3 + j\omega)(j\omega - 1) = 1 \\ \therefore\ A(-\omega^2 + j\omega - 2) + B(-\omega^2 + 5j\omega + 6) + C(-\omega^2 + 2j\omega - 3) = 1 \end{aligned}$$

which implies

$$\begin{aligned} A + B + C &= 0 \\ A + 5B + 2C &= 0 \\ -2A + 6B - 3C &= 1 \end{aligned}$$

and so

$$A = \frac{1}{4},\ B = \frac{1}{12},\ C = \frac{-1}{3}$$

Thus

$$y(t) = \frac{1}{4}\, u(t)\, e^{-3t} - \frac{1}{12}\, u(-t)\, e^{t} - \frac{1}{3}\, u(t)\, e^{-2t}.$$

MATLAB Problem 4

In this problem, I am giving you a data file consisting of several *amplitude modulated* audio signals (similar to AM radio). Let $x_1(t), \ldots, x_9(t)$ denote the 9 audio signals. I have provided the signal

$$r(t) = \sum_{k=1}^{9} \cos(2\pi f_k t)\, x_k(t) \quad \text{where} \quad f_k = 25000\,(k-1).$$

The signals $x_1(t), \ldots, x_9(t)$ can be viewed as the audio content of a radio station, the $\cos(2\pi f_k t)\, x_k(t)$ signals can be viewed as what the radio stations are transmitting, and $r(t)$ can be viewed as the signal your antenna is receiving (since by Maxwell's equations, EM waves are additive). Your goal will be to "tune into" each of the 9 stations and decipher the modulated audio messages.

- Place the files 'mod.mat' and 'noisy.mat' into your MATLAB directory and load them into your workspace using 'load mod.mat;' and 'load noisy.mat;' (warning, these files are quite large at roughly 27MB)
- 'noisy.mat' is the signal

$$\sum_{k=1}^{9} x_k(t).$$

In other words, 'noisy.mat' is what happens when radio stations DO NOT perform modulation. This is akin to multiple people talking to you at once. It is very difficult to pick out any one of the messages, since they are all communicating in the same frequency band.

Output 1: try running 'mod play(noisy);' and describe what you hear.

Note that you will need to use this custom function to play the audio files in this problem (as opposed to the usual 'sound' function).

- We will make use of the fact that

$$x(t)\cos(\omega_0 t) \longleftrightarrow \frac{X(\omega+\omega_0) + X(\omega-\omega_0)}{2}$$

  Each of the signals $x_1(t), \ldots, x_9(t)$ was multiplied by a different *carrier frequency*, and the bandwidth of each signal is smaller than the differences in the carrier frequencies, so

$$R(\omega) = \sum_{k=1}^{9} \big(X_k(\omega - 2\pi f_k) + X_k(\omega - 2\pi f_k)\big)$$

This allows us to "spread out" the signals in the frequency domain. Plot the modulated signal in the frequency domain using:

'Len = length(mod);'

'$Fs = 811025$;'

'$f = Fs * (-\text{Len}/2 : \text{Len}/2 - 1)\,/\,\text{Len}$;'

'Mod Freq = fft(mod);'

'plot(f,abs(fftshift(Mod Freq)));'

Output 2: include well-labeled plots of both the modulated (mod) and unmodulated (noisy) signals in the frequency domain, and explain the differences you notice. You may need to adjust the limits of the axes to better analyze these figures.

- You will be required to submit **Outputs 3 & 4** for any three of the nine audio signals, but I encourage you to try "tuning into" all nine stations. The following steps describe how to recover the kth audio message.

- Filter out all of the modulated signals, except for the kth. You can do this using the provided filter file, e.g.

  'Filtered_Signal = Mod_Freq .$*$ HW3_Filter(f, A, B);'

  You will need to experiment with the values of A and B to correctly filter out the kth signal.

- You will now need to convert this filtered signal back to the time domain, which yields the signal $\cos(2\pi f_k t)\, x_k(t)$. You can do this using

  'filtered_signal = real(ifft(Filtered_Signal));

- You now need to undo the modulation to recover $x_k(t)$. We will be using the fact

$$x(t)\cos^2(\omega_0 t) = x(t)\frac{1+\cos(2\omega_0 t)}{2} \longleftrightarrow \frac{X(\omega)}{2} + \frac{X(\omega+2\omega_0)+X(\omega-2\omega_0)}{4}$$

  So multiplying $\cos(2\pi f_k t)\, x_k(t)$ by $\cos(2\pi f_k t)$ will yield copies of $X(\omega)$ in the frequency domain that are centered at 0 and $\pm 2\pi f_k$. Do this by taking

  't = (0:Len-1) / F_s;'

  'demod_signal = filtered_signal . $*$ $(2 * \cos(2 * \text{pi} * f_k * t))$;'

- Our final step is to filter out the high frequency components so that we are left with our desired message $x_k(t)$. We can do this by sending $x(t)\cos^2(\omega_0 t)$ through a low-pass filter that zeros out the $X_k(\omega \pm 2\omega_k)$ terms but allows the $X_k(\omega)$ term to pass through.

  'Demod_Signal = fft(demod);'

  'Message = Demod_Signal .$*$ HW3_Filter(f, A, B);'

  You will need to experiment with the values of A and B to filter out the high frequency terms.

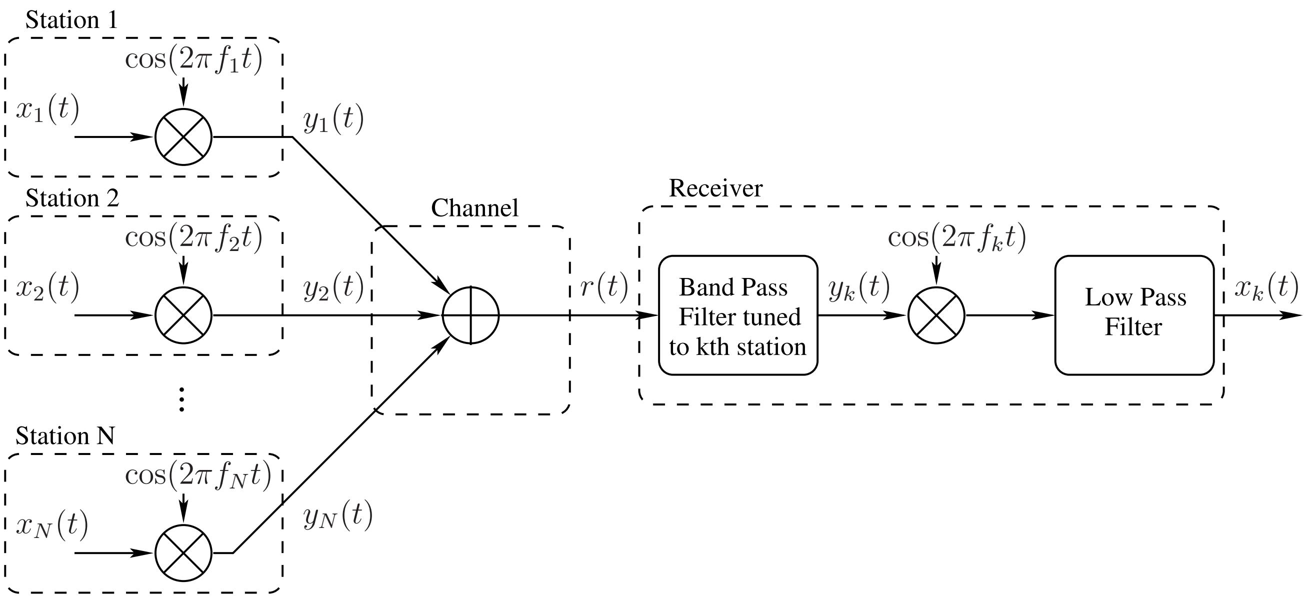
  Output 3: include a well-labeled plot of the message in the frequency domain.

  Now we convert back to the time domain and play using the provided function:

  'message = real(ifft(Message));'

  'mod_play(message);'

  Output 4: What is the kth audio message?

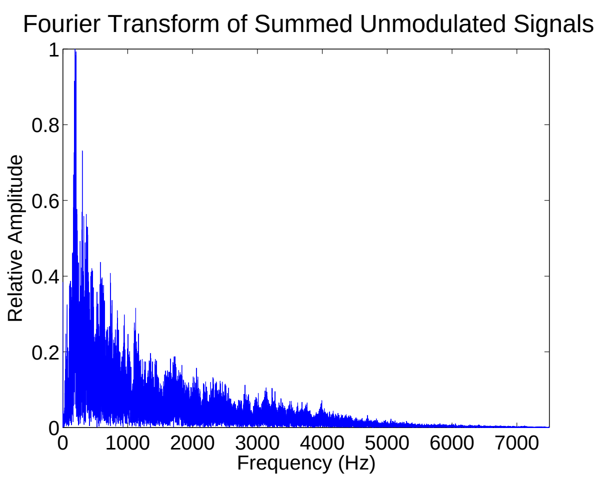


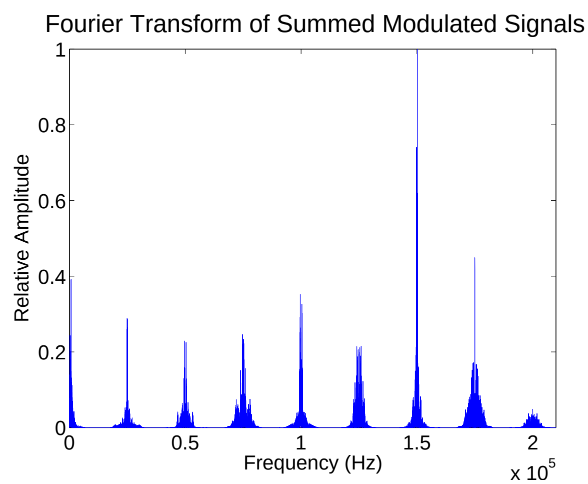
A block diagram of the basic amplitude modulation scheme used in this problem. Modulation allows for concurrent communication by dividing up portions of the frequency spectrum.

Solution:

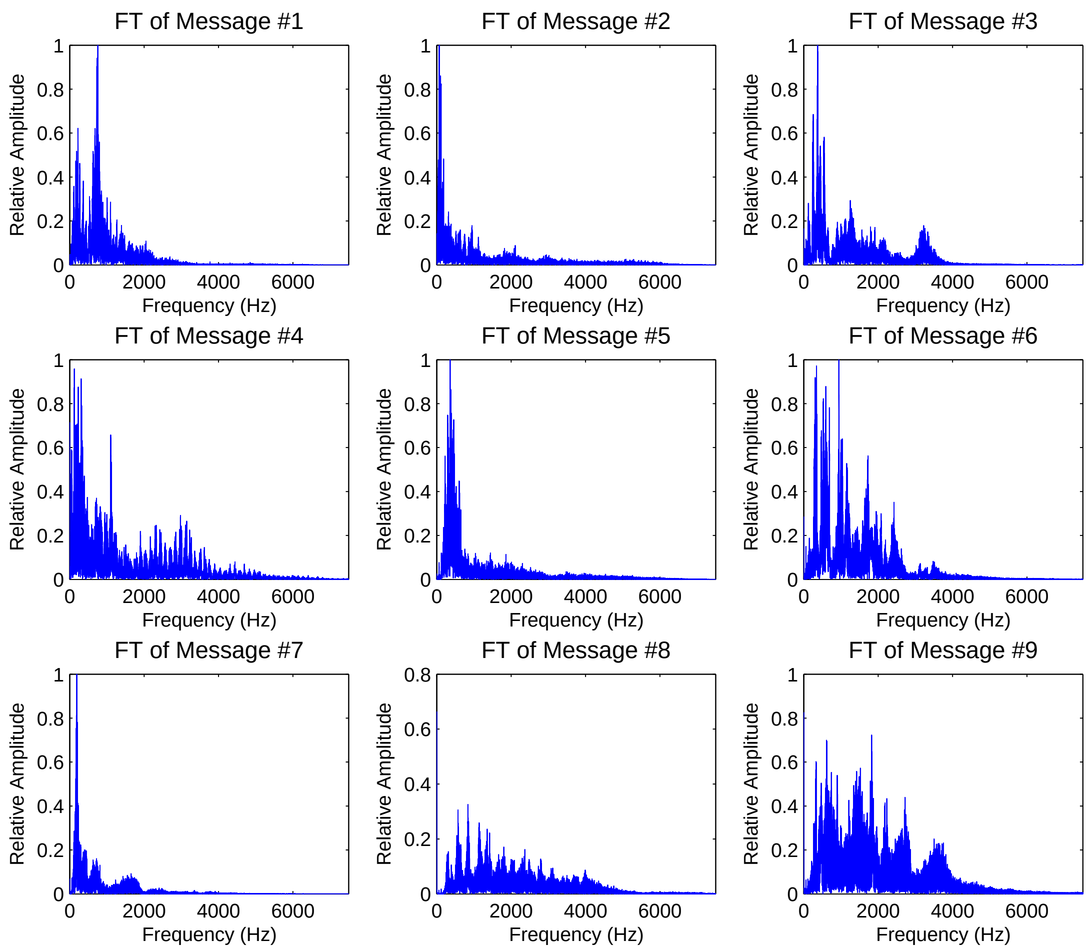
See ECE45_MATLAB3.m for implementation details.

When running 'mod_play(noisy);' it sounds like 9 people talking at once. This is because the 9 messages $x_1(t), \ldots, x_9(t)$ are simply summed in the noisy file. This results in a jumbled audio file in which it is difficult to pick out any one message.





In the modulated figure, the signals are spread out in the frequency domain. The kth peak corresponds to the (shifted) Fourier transform of the kth message. Since the signals are spread out in frequency, we can filter out undesired signals. Once the undesired signals are filtered out, we can demodulate by multiplying by the carrier signal and filtering.



The Fourier transforms of the messages.

1. "I'll be back." - The Terminator in *The Terminator*, 1984.

2. "The ring must be destroyed!" - Gandalf the Grey in *The Lord of the RIngs: The Fellowship of the Ring*, 2001.

3. "Bueller? Bueller?" - Ben Stein in *Ferris Bueller's Day Off*, 1986.

4. "You have failed me for the last time." - Darth Vader in *Star Wars: The Empire Strikes Back*, 1980.

5. "Mmm. This is a tasty burger." - Jules Winnfield in *Pulp Fiction*, 1994.

6. "Game over, man. It's game over!" - Private Hudson in *Aliens*, 1986.

7. "I'm sorry, Dave. I'm afraid I can't do that." - HAL 9000 in *2001: A Space Odyssey*, 1968.

8. "How am I gonna generated that kind of power? It can't be done." - Doc Brown in *Back to the Future*, 1985.

9. "Who died and made you Einstein?" - Valentine McKee in *Tremors*, 1990.