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ECE 45 Homework 3

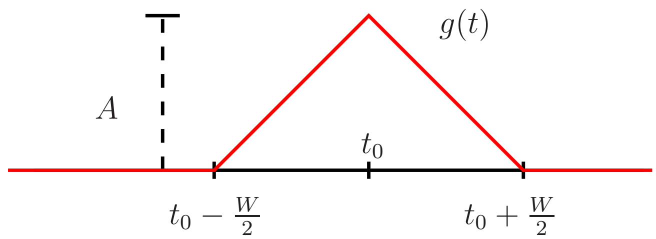
Problem 3.1 Calculate the Fourier transform of the function

$$\Delta(t) = \begin{cases} 1 - 2|t| & |t| \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

Use the statement of **Problem 3.2** to verify your answer. **Note:** the function $\Delta(t)$ is sometimes called the *unit triangle function*, as it a triangular pulse with height 1, width 1, and is centered at 0.

Hint: Recall the trig identity $1 - \cos(2x) = 2\sin^2(x)$. Also, the time-reversal property can be helpful.

Problem 3.2 Let A, W, and t_0 be real numbers such that $A, W > 0$, and suppose that $g(t)$ is given by



Show the Fourier transform of $g(t)$ is equal to

$$\frac{AW}{2} \operatorname{sinc}^2 (W\omega/4)\, e^{-j\omega t_0}$$

using the results of Problem 3.1 and the properties of the Fourier transform.

Hint: You do NOT have to re-integrate, this should only take a few lines.

Problem 3.3 Find the Fourier transform of the function

$$x(t) = \begin{cases} 1 & \text{if } 1 \leq |t| \leq 3 \\ -1 & \text{if } |t| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Hint: Recall rectangle functions to reduce amount of integration.

Problem 3.4 Find the inverse Fourier transform of the function

$$F(\omega) = \frac{12 + 7j\omega - \omega^2}{(\omega^2 - 2j\omega - 1)(-\omega^2 + j\omega - 6)}$$

Hint: Use Partial fractions.

Problem 3.5 Suppose a function $f(t)$ has Fourier transform

$$F(\omega) = 2\pi j\omega e^{-|\omega|}.$$

Is $f(t)$ purely real? Is $f(t)$ purely imaginary? Is $f(t)$ even? Is $f(t)$ odd? What is $f(0)$? Calculate $f(t)$ and verify these properties.

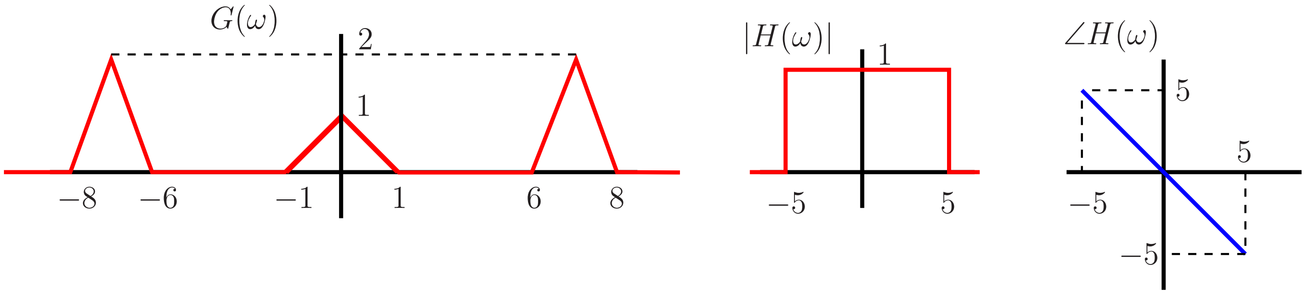
Hint: For finding the inverse Fourier transform, use this property: Let G(w) = 2πe^(−|ω|) and F(ω) = jωG(ω) and use the derivative property.

Problem 3.6 Suppose $x(t)$ is the input to an LTI system with transfer function $H(\omega)$, and $y(t)$ is the output of this system, where

$$x(t) = e^{-|t|}\cos(At) \text{ and } H(\omega) = 1 + e^{-j\omega} + e^{-3j\omega}.$$

Find a real number $A > 0$ such that $y(0) = 1$. Is your answer unique?

Problem 3.7 Suppose $g(t)$ is the input to an LTI system with transfer function $H(\omega)$, and $G(\omega)$ is the Fourier transform of $g(t)$. Find the output of the system $y(t)$.



Problem 3.8 Recall the *unit step function* $u(t)$ is given by $u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$

Suppose we have a system for which the output $y(t)$ is

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

when the input is $x(t)$. Find $y(t)$ and its Fourier transform $Y(\omega)$ when the input is

$$x(t) = u(t+1) - 2u(t-1) + u(t-3).$$

Problem 3.9 An LTI system has impulse response $h(t) = e^{-3t}u(t)$. What was the input $x(t)$, when the output is $e^{-3t}u(t) - e^{-4t}u(t)$?

Problem 3.10 Let $x(t) = u(t)e^{-3t}$. Find $y(t)$ when

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} - 2y(t) = x(t).$$

Hint: Use Partial Fractions.

MATLAB Problem 4

You MUST attach your printed out code to this assignment

In this problem, I am giving you a data file consisting of several *amplitude modulated* audio signals (similar to AM radio). Let $x_1(t), \ldots, x_9(t)$ denote the 9 audio signals. I have provided the signal

$$r(t) = \sum_{k=1}^{9} \cos(2\pi f_k t)\, x_k(t) \quad \text{where} \quad f_k = 25000\,(k-1).$$

The signals $x_1(t), \ldots, x_9(t)$ can be viewed as the audio content of a radio station, the $\cos(2\pi f_k t)\, x_k(t)$ signals can be viewed as what the radio stations are transmitting, and $r(t)$ can be viewed as the signal your antenna is receiving (since by Maxwell's equations, EM waves are additive). Your goal will be to "tune into" each of the 9 stations and decipher the modulated audio messages.

- Place the files 'mod.mat' and 'noisy.mat' into your MATLAB directory and load them into your workspace using 'load mod.mat;' and 'load noisy.mat;' (warning, these files are quite large at roughly 27MB)
- 'noisy.mat' is the signal

$$\sum_{k=1}^{9} x_k(t).$$

In other words, 'noisy.mat' is what happens when radio stations DO NOT perform modulation. This is akin to multiple people talking to you at once. It is very difficult to pick out any one of the messages, since they are all communicating in the same frequency band.

Output 1: try running 'mod_play(noisy);' and describe what you hear.

Note that you will need to use this custom function to play the audio files in this problem (as opposed to the usual 'sound' function).

- We will make use of the fact that

$$x(t)\cos(\omega_0 t) \longleftrightarrow \frac{X(\omega + \omega_0) + X(\omega - \omega_0)}{2}$$

Each of the signals $x_1(t), \ldots, x_9(t)$ was multiplied by a different *carrier frequency*, and the bandwidth of each signal is smaller than the differences in the carrier frequencies, so

$$R(\omega) = \sum_{k=1}^{9} \big(X_k(\omega - 2\pi f_k) + X_k(\omega - 2\pi f_k)\big)$$

This allows us to "spread out" the signals in the frequency domain. Plot the modulated signal in the frequency domain using:

'Len = length(mod);'

'$Fs = 811025$;'

'$f = Fs * (-\text{Len}/2 : \text{Len}/2 - 1) \,/\, \text{Len}$;'

'Mod_Freq = fft(mod);'

'plot(f,abs(fftshift(Mod_Freq)));'

Output 2: include well-labeled plots of both the modulated (mod) and unmodulated (noisy) signals in the frequency domain, and explain the differences you notice. You may need to adjust the limits of the axes to better analyze these figures.

- You will be required to submit **Outputs 3 & 4** for any three of the nine audio signals, but I encourage you to try "tuning into" all nine stations. The following steps describe how to recover the kth audio message.

- Filter out all of the modulated signals, except for the kth. You can do this using the provided filter file, e.g.

  'Filtered_Signal = Mod_Freq .* HW3_Filter(f, A, B);'

  You will need to experiment with the values of A and B to correctly filter out the kth signal.

- You will now need to convert this filtered signal back to the time domain, which yields the signal $\cos(2\pi f_k t)\, x_k(t)$. You can do this using

  'filtered_signal = real(ifft(Filtered_Signal));

- You now need to undo the modulation to recover $x_k(t)$. We will be using the fact

$$x(t)\cos^2(\omega_0 t) = x(t)\frac{1+\cos(2\omega_0 t)}{2} \longleftrightarrow \frac{X(\omega)}{2} + \frac{X(\omega+2\omega_0)+X(\omega-2\omega_0)}{4}$$

  So multiplying $\cos(2\pi f_k t)\, x_k(t)$ by $\cos(2\pi f_k t)$ will yield copies of $X(\omega)$ in the frequency domain that are centered at 0 and $\pm 2\pi f_k$. Do this by taking

  't = (0:Len-1) / F_s;'

  'demod_signal = filtered_signal . $*$ $(2 * \cos(2 * \text{pi} * f_k * t))$;'

- Our final step is to filter out the high frequency components so that we are left with our desired message $x_k(t)$. We can do this by sending $x(t)\cos^2(\omega_0 t)$ through a low-pass filter that zeros out the $X_k(\omega \pm 2\omega_k)$ terms but allows the $X_k(\omega)$ term to pass through.

  'Demod_Signal = fft(demod);'

  'Message = Demod_Signal .* HW3_Filter(f, A, B);'

  You will need to experiment with the values of A and B to filter out the high frequency terms.

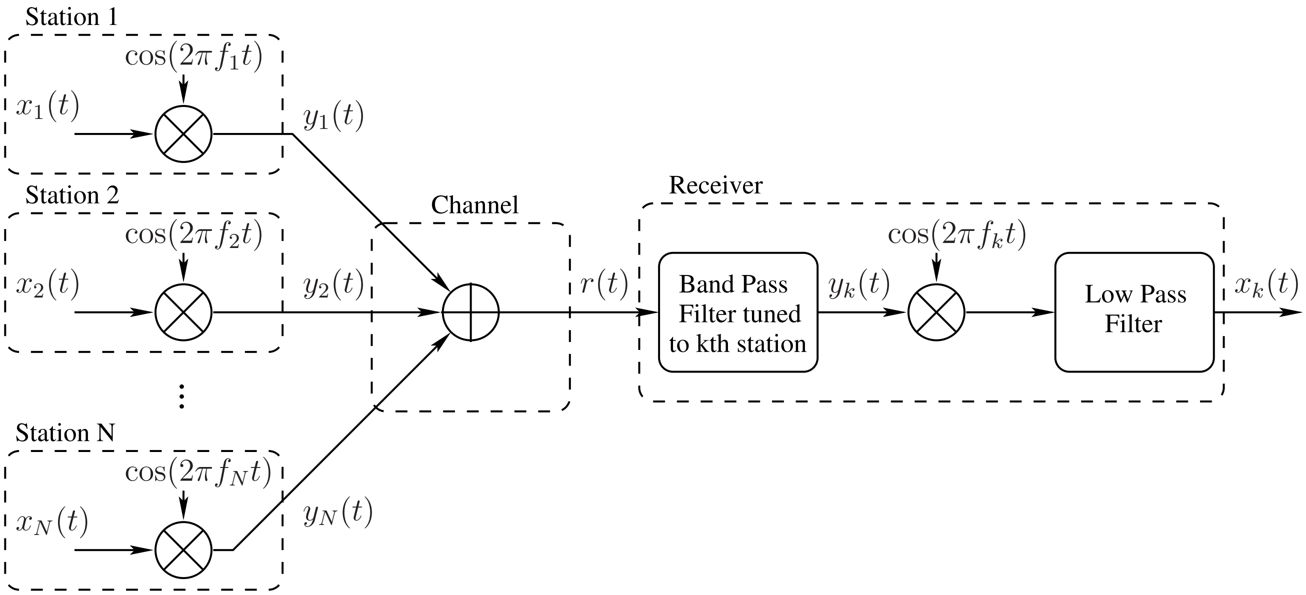
  Output 3: include a well-labeled plot of the message in the frequency domain.

  Now we convert back to the time domain and play using the provided function:

  'message = real(ifft(Message));'

  'mod_play(message);'

  Output 4: What is the kth audio message?



A block diagram of the basic amplitude modulation scheme used in this problem. Modulation allows for concurrent communication by dividing up portions of the frequency spectrum.