ECE 45 Homework 2

Problem 2.1

Are the following systems linear? Are they time invariant?

(a) $x(t) \longrightarrow [\text{ System (a) }] \longrightarrow 2x(t-3)$

(b) $x(t) \longrightarrow [\text{ System (b) }] \longrightarrow x(t)+t$

(c) $x(t) \longrightarrow [\text{ System (c) }] \longrightarrow (x(t)+1)^2$

(d) $x(t) \longrightarrow [\text{ System (d) }] \longrightarrow \cos(x(t))$

(e) $x(t) \longrightarrow [\text{ System (e) }] \longrightarrow \int_{-\infty}^{t} x(\tau)\, d\tau$

(f) $x(t) \longrightarrow [\text{ System (f) }] \longrightarrow t.$

i.e. the term on the right is the output when the input is $x(t)$. Plot the output, in each case, when

$$x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Problem 2.2

Find the Bode plot (magnitude and phase) and label all critical points of the transfer function

$$H(\omega) = \frac{225\,(\omega^2 - 2500j\omega - 10^6)\,(1+10^{-6}j\omega)}{9\,\left(\frac{\omega^2}{9} - \frac{20000j\omega}{3} - 10^8\right)\,(25+5j\omega)}.$$

MATLAB: Include MATLAB plots of the magnitude and angle of this function on the same scale. How does the Bode plot approximation compare to the actual function?

Tips: ‘logspace(a,b,n)’ generates n points between decades 10^a and 10^b.

‘semilogx(x,y)’ plots y versus x with a log scale on the x axis.

‘log10(x)’ returns $\log_{10}(x)$, whereas ‘log(x)’ returns the natural log.

Problem 2.3

Are the following functions periodic? If so, find the period and fundamental frequency.

(a) $f_1(t) = \cos^2(10t)$

(b) $f_2(t) = \sum_{n=-\infty}^{\infty} x(t-n)$, where $x(t)=t$ for $0 \le t < 1$ and $x(t)=0$ otherwise.

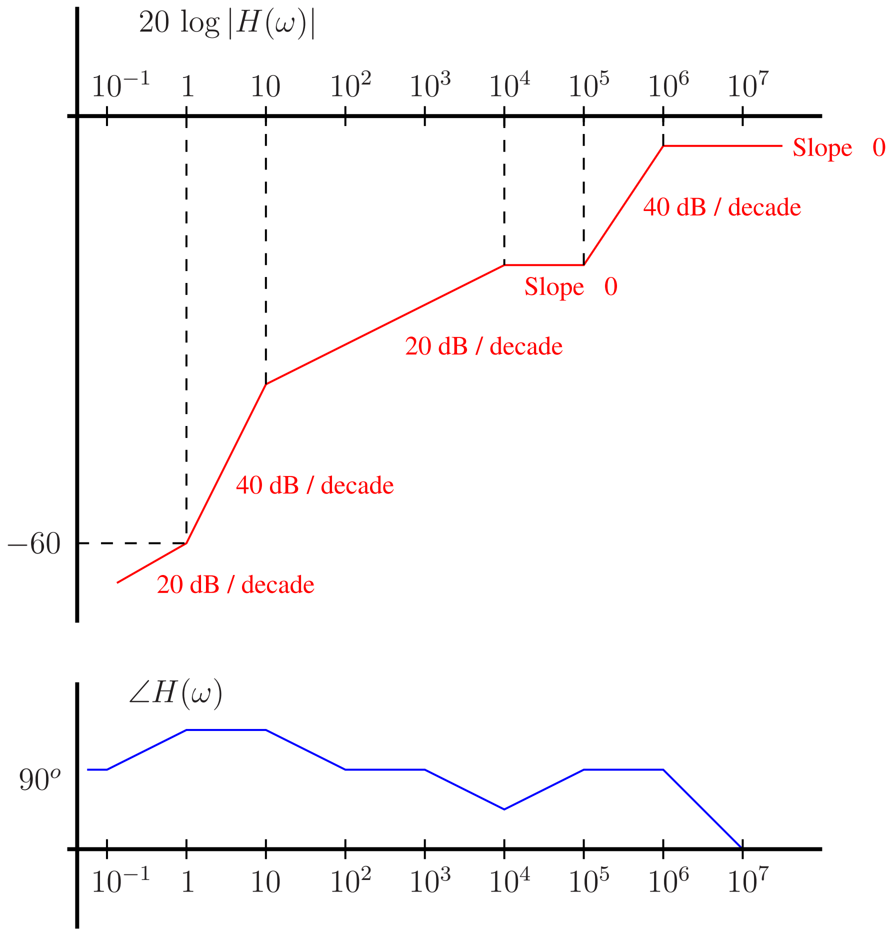
(c) $f_3(t) = \tan(t)$

(d) $f_4(t) = \cos(t) + \cos(\pi t)$

Please report any typos/errors to j2connelly@ucsd.edu

Problem 2.4

Suppose $H(\omega)$ in the Bode Plot given below is the transfer function of an LTI system. Determine $H(\omega)$, assuming the plot uses the linear approximation techniques from class, and use the approximations from the Bode plot to determine the output of the system when the input is



(a) $x_1(t) = \cos(0.1t)$

(b) $x_2(t) = \cos(8t)$

(c) $x_3(t) = \cos(30t)$

(d) $x_4(t) = \sin(2000t)$

(e) $x_5(t) = \cos(50000t)$

Problem 2.5

Suppose $f(t)$ is a periodic function with period $T = 2$ and Fourier series components

$$F_n = \frac{1}{2n^2} \quad \text{for all integers } n \neq 0 \text{ and } F_0 = 1.$$

$f(t)$ is the input to an LTI system with transfer function

$$H(\omega) = \cos(\omega) + j\sin(\omega).$$

Find the output, $y(t)$, of the system in terms of only real numbers (all imaginary components should cancel).

Problem 2.6

Find the Fourier series components F_n of $f(t) = \sin^4(t)$.

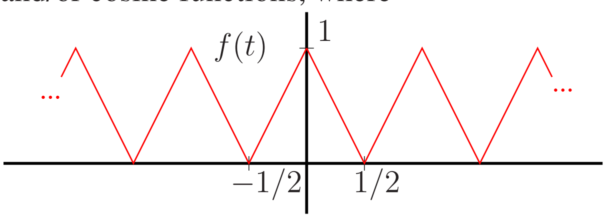
Problem 2.7

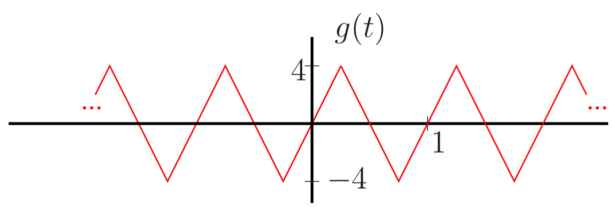
Determine the Fourier series coefficients F_n and find the average power in a period of the function:

$$\sum_{n=-\infty}^{\infty} x(t-4n) \quad \text{where} \quad x(t) = \begin{cases} 1 & 0 \le t < 2 \\ 2 & 2 \le t < 4 \end{cases}$$

Problem 2.8

Find the Fourier series components of $f(t)$ and $g(t)$ and write $f(t)$ and $g(t)$ as purely real sums of sine and/or cosine functions, where





Hint: After calculating the components of $f(t)$, use the properties to find the components of $g(t)$.

Problem 2.9

Find the Fourier series components of $f(t)$. Using the properties of the Fourier series, find the Fourier series coefficients of $g(t)$, where

$$f(t) = 2 + \sum_{n=-\infty}^{\infty} x(t-4n) \quad \text{and} \quad g(t) = \sum_{n=-\infty}^{\infty} y(t-4n)$$

$$x(t) = \begin{cases} t-2 & \text{when } 0 \le t < 4 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad y(t) = \begin{cases} t^2/4 & \text{when } -2 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Hint: Write $x(t)$ in terms of $y(t)$ and write $f(t)$ in terms of $g(t)$.

Problem 2.10

Suppose $f(t)$ is the input to an LTI system with transfer function $H(\omega)$, where

$$f(t) = |\sin(t)| \quad \text{and} \quad H(\omega) = 1 - 4\left(\frac{\omega}{2\pi}\right)^2$$

Find the output $y(t)$ and write it as a purely real sum of sines and/or cosines.

MATLAB Problem 3

In this problem, I am providing you with four noisy vectors of length $N = 11613$. Embedded in one of these vectors is a famous quote from a former politician. Your goal will be to determine which of these four vectors contains the audio signal (and determine which three are purely noise) and decode the audio signal.

- Place the files 'one.mat', ..., 'four.mat' in your MATLAB directory.
- Run the commands 'load one.mat;', ... 'load four.mat;' to load the vectors.
- Declare variables '$F_s = 11025$; and '$t = (0 : N - 1)/F_s$;'

  (F_s is the frequency the audio message was sampled at, and t is an array of time (in seconds) you can use to plot. The sampling frequency essentially tells us that all frequencies present in the signal are at most $\frac{F_s}{2}$. We will learn more about this later)
- Try plotting 'one', 'two', 'three', and 'four' and running 'sound(one, F_s);',..., 'sound(four, F_s);'

  (warning the sound will be unpleasant).

  You will likely be unable to determine which signal contains the message by examining these signals in the time domain.
- Fortunately for us, the noise I used only has frequencies outside of the range of frequencies in the audio clip. So while the signals are garbled in the time domain, we may be able to decipher some information in the frequency domain.
- Try plotting the magnitude of each signal in the frequency domain.

  To do this, run the following commands:

  '$f = (-F_s/2 : F_s/(N-1) : F_s/2)$;'

  (this creates a length-N array whose entries range from $\frac{-F_s}{2}$ to $\frac{F_s}{2}$ and increment by $\frac{F_s}{N-1}$)

  'One = fft(one);'

  (this creates an array of length N that represents one in the frequency domain)

  'plot(f, abs(fftshift(One)));'

  If they still all look similar to you, try plotting the logarithm of the absolute value of the magnitude of the signal in the frequency domain.

  'plot(f, $\log$(abs(fftshift(One))));'

  By doing this with all four signals, you should notice that one of the signals contains frequency components that the other three do not.
- Once you have identified the signal with the audio embedded in it, you will need to filter out the noise. To do this, we can use an ideal high, low, or band-pass filter, and we can work entirely in the frequency domain.

  Let 'X' denote the frequency domain array (i.e. $X = \text{fft}(x)$) of the signal with the audio embedded in it. I have provided you with a function for an ideal band-pass filter (HW2_Filter.m), which you can use for filtering by doing something like:

  '$Z = X .*$ HW2_Filter(f, A, B);'

  You will need to experiment with the values of A and B.

  Alternatively, you can figure out another way to filter out the noise, if you'd like.

- Finally, once you have filtered out the noise in the frequency domain, you need to convert Z back into the time domain. You can do this by taking:

  '$z = \text{real}(\text{ifft}(Z))$;'

  'sound(z, F_s);'

  If you implemented the filtering correctly, you should hear the audio.

Submit the following:

- which signal has the audio embedded in it
- the contents of audio message
- a plot of the magnitude frequency domain representation of the unfiltered signal (with proper labeling, of course), and a plot of the magnitude of the frequency domain representation of the filtered audio message.