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ECE 45 Homework 2 Solutions

Problem 2.1

Are the following systems linear? Are they time invariant?

(a) $x(t) \longrightarrow$ [System (a)] $\longrightarrow 2x(t-3)$

(b) $x(t) \longrightarrow$ [System (b)] $\longrightarrow x(t) + t$

(c) $x(t) \longrightarrow$ [System (b)] $\longrightarrow (x(t)+1)^2$

(d) $x(t) \longrightarrow$ [System (d)] $\longrightarrow \cos(x(t))$

(e) $x(t) \longrightarrow$ [System (e)] $\longrightarrow \int_{-\infty}^{t} x(\tau)\, d\tau$

(f) $x(t) \longrightarrow$ [System (f)] $\longrightarrow t.$

i.e. the term on the right is the output when the input is $x(t)$. Plot the output, in each case, when

$$x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

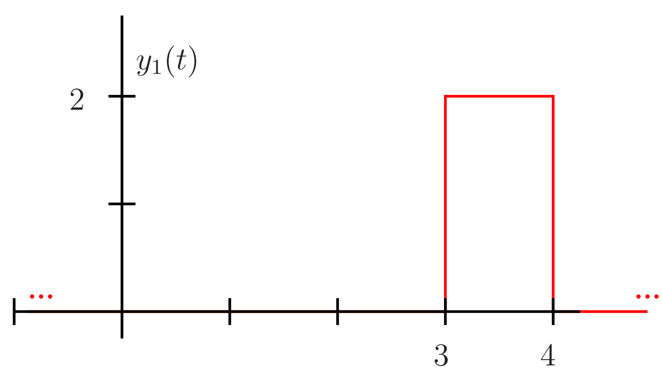
Solution:

(a) For any functions $x_1(t), x_2(t)$ and real numbers a, b, t_1, t_2, we have

$$ax_1(t-t_1) + bx_2(t-t_2) \longrightarrow \text{[System (a)]} \longrightarrow a2x_1(t-t_1-3) + b2x_2(t-t_2-3)$$

Thus the system is both linear and time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$, the output is $\begin{cases} 2 & \text{if } 3 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$



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(b) For any function $x(t)$ and any real number a, we have

$$ax(t) \longrightarrow [\text{ System (b) }] \longrightarrow (ax(t)+t) \neq a\,(x(t)+t)$$

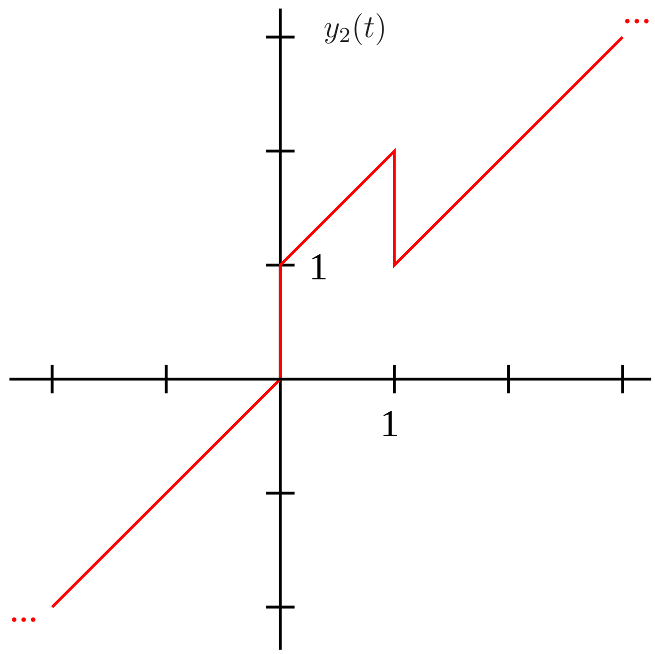
so the system is not linear.

For any real number t_0, we have

$$x(t-t_0) \longrightarrow [\text{ System (b) }] \longrightarrow (x(t-t_0)+t) \neq (x(t-t_0)+t-t_0)$$

so the system is not time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$, the output is $\begin{cases} t+1 & \text{if } 0 \leq t \leq 1 \\ t & \text{otherwise} \end{cases}$



(c) For any function $x(t)$, we have

$$x(t)-x(t)=0 \;\longrightarrow\; [\text{ System (c) }] \;\longrightarrow\; 1 \neq 0$$

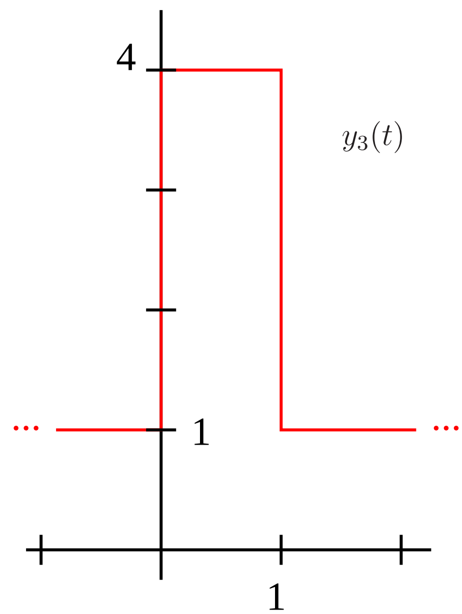
so the system is not linear.

For any real number t_0, we have

$$x(t-t_0) \;\longrightarrow\; [\text{ System (c) }] \;\longrightarrow\; x(t-t_0)+1$$

so the system is time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$, the output is $\begin{cases} 4 & \text{if } 0 \leq t \leq 1 \\ 1 & \text{otherwise} \end{cases}$



(d) For any functions $x(t)$, we have

$$x(t) - x(t) = 0 \longrightarrow [\text{ System (d) }] \longrightarrow \cos(0) = 1 \neq 0$$

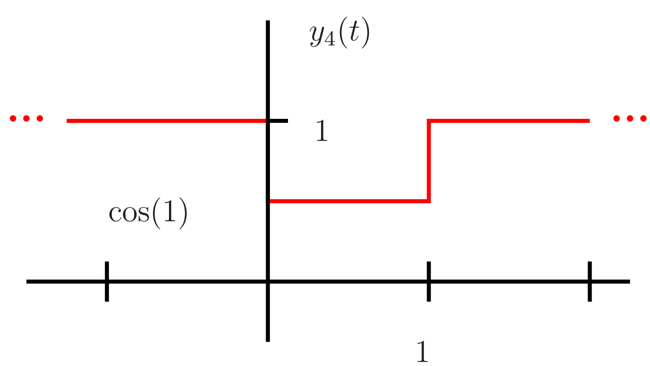
so the system is not linear.

For any real number t_0, we have

$$x(t - t_0) \longrightarrow [\text{ System (d) }] \longrightarrow \cos(x(t - t_0))$$

so the system is time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$, the output is $\begin{cases} \cos(1) & \text{if } 0 \le t \le 1 \\ 1 & \text{otherwise} \end{cases}$



(e) Note that for any function $x(t)$ and any real number c, by letting $z = \tau - c$, we have

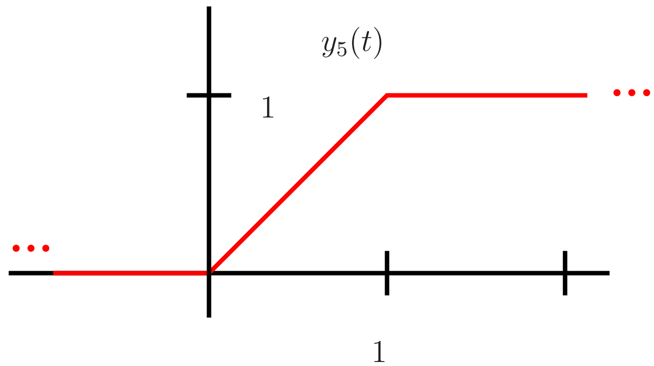
$$\int_{-\infty}^{t} x(\tau - c)\, d\tau = \int_{-\infty}^{t-c} x(z)\, dz$$

For any functions $x_1(t), x_2(t)$ and real numbers a, b, t_1, t_2, we have

$$\begin{aligned} ax_1(t - t_1) + bx_2(t - t_2) \longrightarrow [\text{ System (e) }] \longrightarrow & \; a \int_{-\infty}^{t} x_1(\tau - t_1)\, d\tau + b \int_{-\infty}^{t} x_2(\tau - t_1)\, d\tau \\ & = a \int_{-\infty}^{t-t_1} x_1(z)\, dz + b \int_{-\infty}^{t-t_2} x_2(z)\, dz. \end{aligned}$$

Thus the system is both linear and time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$, the output is $\begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } 0 \le t \le 1 \\ 1 & \text{if } t > 1 \end{cases}$



(f) For any functions $x(t)$, we have

$$x(t) - x(t) = 0 \longrightarrow [\text{ System (f) }] \longrightarrow t \neq 0$$

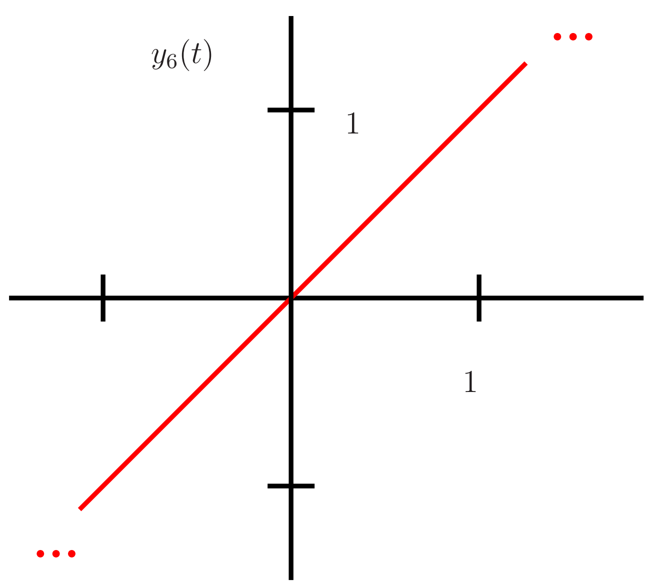
so the system is not linear.

For any real number t_0, we have

$$x(t - t_0) \longrightarrow [\text{ System (f) }] \longrightarrow t \neq t - t_0$$

so the system is not time invariant.

When $x(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$, the output is t



Problem 2.2

Find the Bode plot (magnitude and phase) and label all critical points of the transfer function

$$H(\omega) = \frac{225\left(\omega^2 - 2500j\omega - 10^6\right)\left(1 + 10^{-6}j\omega\right)}{9\left(\frac{\omega^2}{9} - \frac{20000j\omega}{3} - 10^8\right)\left(25 + 5j\omega\right)}.$$

MATLAB: Include MATLAB plots of the magnitude and angle of this function on the same scale. How does the Bode plot approximation compare to the actual function?

Tips: 'logspace(a,b,n)' generates n points between decades 10^a and 10^b.

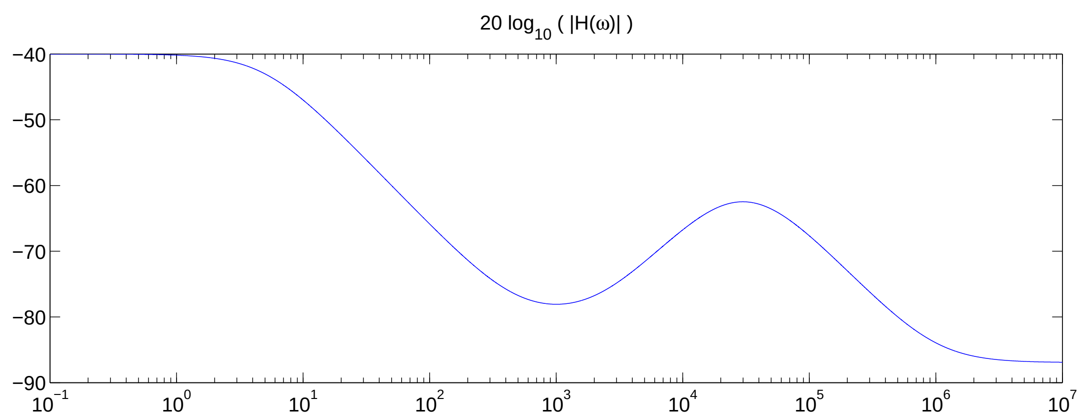
'semilogx(x,y)' plots y versus x with a log scale on the x axis.

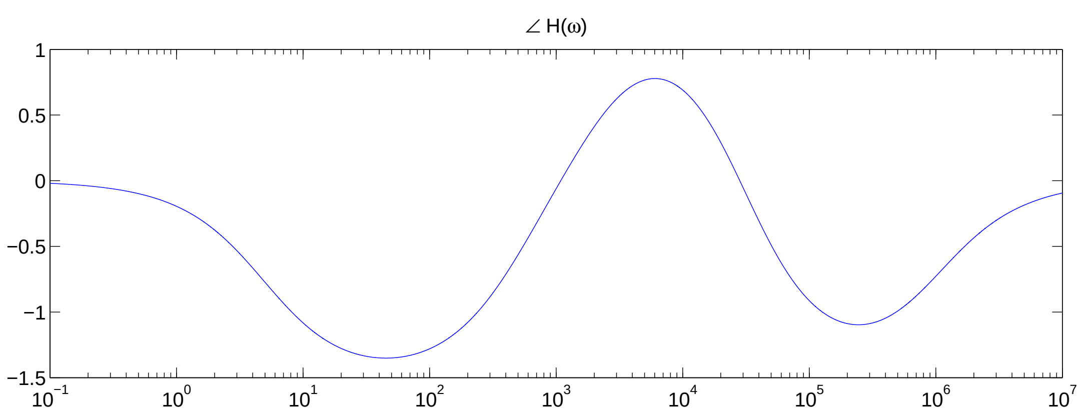
'log10(x)' returns $\log_{10}(x)$, whereas 'log(x)' returns the natural log.

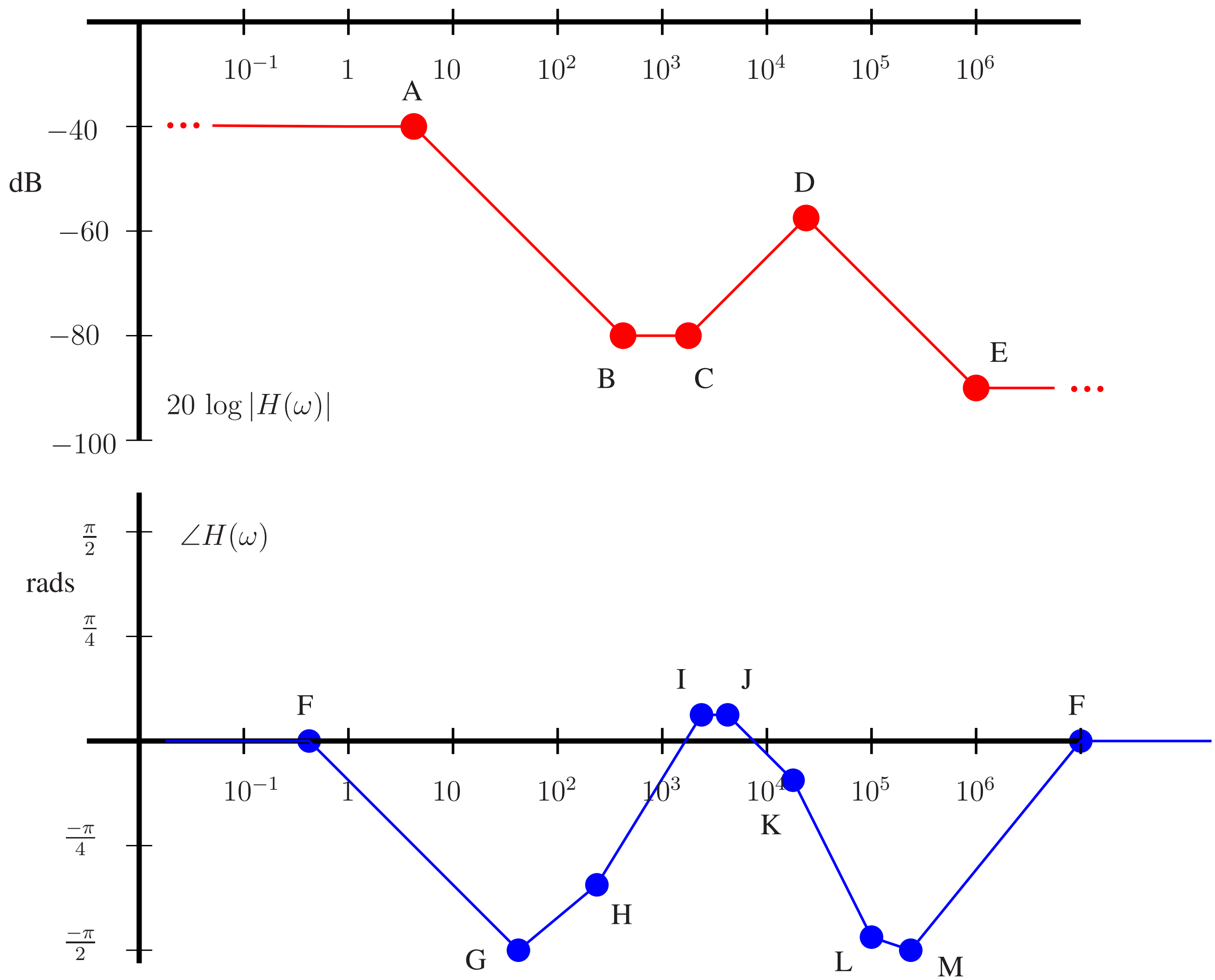
Solution:

We first write $H(\omega)$ in standard form:

$$H(\omega) = \frac{\left(1 + \frac{j\omega}{2000}\right)\left(1 + \frac{j\omega}{500}\right)\left(1 + \frac{j\omega}{10^6}\right)}{100\left(1 + \frac{j\omega}{30000}\right)^2\left(1 + \frac{j\omega}{5}\right)}.$$







Critical Points:

A) $20\log(|H(5)|) = -40$

B) $20\log(|H(500)|) = -80$

C) $20\log(|H(2000)|) = -80$

D) $20\log(|H(30000)|) = -80 + 20\log(15)$

E) $20\log(|H(10^6)|) = -120 + 20\log(45)$

F) $\angle H(1/5) = 0$

G) $\angle H(50) = -\pi/2$

H) $\angle H(200) = (\log(2) - 1)\pi/2$

I) $\angle H(3000) = \log(3)\pi/2$

J) $\angle H(5000) = \log(3)\pi/2$

K) $\angle H(20000) = (\log(3) - \log(2))\pi/2$

L) $\angle H(100000) = (\log(3) - 1)\pi/2$

M) $\angle H(200000) = (2\log(3) - \log(2) - 2)\pi/4$

N) $\angle H(10^7) = 0.$

Problem 2.3

Are the following functions periodic? If so, find the period and fundamental frequency.

(a) $f_1(t) = \cos^2(10t)$

(b) $f_2(t) = \sum_{n=-\infty}^{\infty} x(t-n)$, where $x(t) = t$ for $0 \leq t < 1$ and $x(t) = 0$ otherwise.

(c) $f_3(t) = \tan(t)$

(d) $f_4(t) = \cos(t) + \cos(\pi t)$

Solution:

(a) $f_1(t) = \frac{1+\cos(20t)}{2}$, so $\omega_0 = 20$ and $T = \pi/10$.

(b) Note that for any t we have

$$f_2(t-1) = \sum_{n=-\infty}^{\infty} x(t-n-1) = \sum_{k=-\infty}^{\infty} x(t-k) = f_2(t)$$

so $f_2(t)$ is periodic, its period is 1, and $\omega_0 = 2\pi$.

(c) For all t we have

$$f_3(t-\pi) = \frac{\sin(t-\pi)}{\cos(t-\pi)} = \frac{-\sin(t)}{-\cos(t)} = \tan(t) = f_3(t)$$

so $f_3(t)$ is periodic and $T = \pi$, so $\omega_0 = 2$.

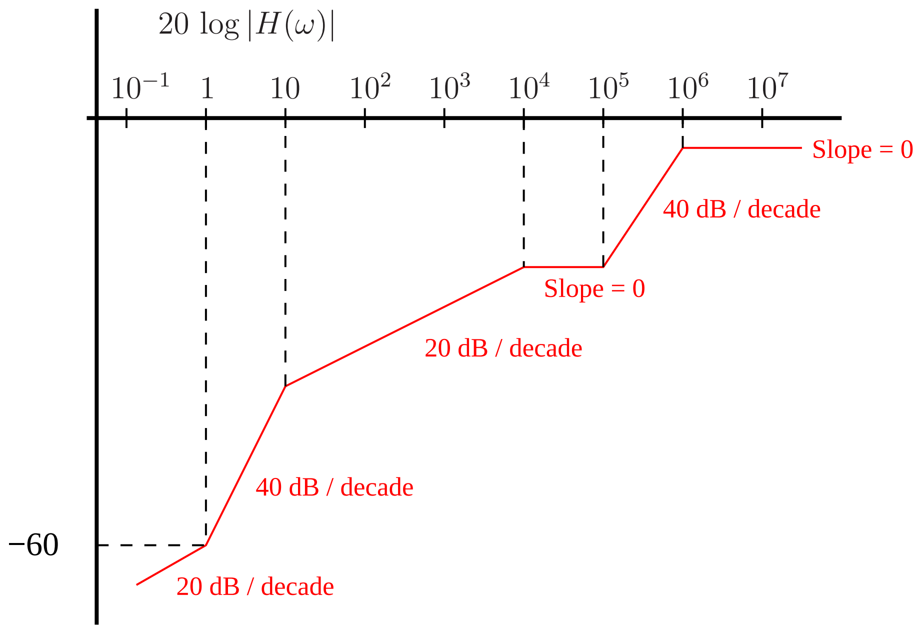
(d) Note that $\cos(t)$ starts/ends a cycle at ..., $6, 4, 2, 0, 2\pi, 4\pi, 6\pi$, ..., i.e. its period is 2π.

$\cos(\pi t)$ starts/ends a cycle at ..., $-6, -2, 0, 2, 4, 6$, ..., i.e. its period is 2.

There is no value of t where both $\cos(t)$ and $\cos(\pi t)$ start/end a cycle, since $2n\pi$ is never an integer! Although this function appears to repeats itself, it has no period, so it is not periodic.

Problem 2.4

Suppose $H(\omega)$ in the Bode Plot given below is the transfer function of an LTI system. Determine $H(\omega)$, assuming the plot uses the linear approximation techniques from class, and use the approximations from the Bode plot to determine the output of the system when the input is



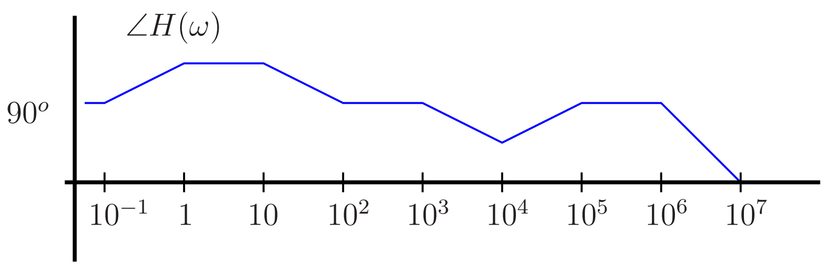
(a) $x_1(t) = \cos(0.1t)$

(b) $x_2(t) = \cos(8t)$

(c) $x_3(t) = \cos(30t)$

(d) $x_4(t) = \sin(2000t)$

(e) $x_5(t) = \cos(50000t)$



Solution:

We have

$$H(\omega) = \frac{(j\omega)(1+j\omega)\left(1+\frac{j\omega}{10^5}\right)^2}{10^3\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^4}\right)\left(1+\frac{j\omega}{10^6}\right)^2}$$

(a) $20\log(|H(0.1)|) = -80$ and $\angle H(0.1) = \pi/2$, so $y_1(t) = 10^{-4}\cos(0.1t + \pi/2)$.

(b) $20\log(|H(8)|) = -60 + 40\log(8)$ and $\angle H(8) = 3\pi/4$, so $y_2(t) = \frac{8}{125}\cos(8t + 3\pi/4)$.

(c) $20\log(|H(30)|) = -20 + 20\log(3)$ and $\angle H(30) = \frac{3\pi}{4} - \frac{\pi}{4}\log(3)$,
so $y_3(t) = -\frac{3}{10}\cos(30t + \frac{3\pi}{4} - \frac{\pi}{4}\log(3))$.

(d) $20\log(|H(2000)|) = -20 + 20\log(200)$ and $\angle H(2000) = \frac{\pi}{2} - \frac{\pi}{4}\log(2)$,
so $y_4(t) = 20\cos(2000t - \frac{\pi}{4}\log(2))$.

(e) $20\log(|H(50000)|) = 40$ and $\angle H(50000) = \frac{\pi}{4} + \frac{\pi}{4}\log 5$,
so $y_5(t) = 100\cos(50000t + \frac{\pi}{4} + \frac{\pi}{4}\log 5)$.

Problem 2.5

Suppose $f(t)$ is a periodic function with period $T = 2$ and Fourier series components

$$F_n = \frac{1}{2n^2} \quad \text{for all integers } n \neq 0 \text{ and } F_0 = 1.$$

$f(t)$ is the input to an LTI system with transfer function

$$H(\omega) = \cos(\omega) + j\sin(\omega).$$

Find the output, $y(t)$, of the system in terms of only real numbers (all imaginary components should cancel).

Solution:

Since $f(t)$ is periodic with period $T = 2$ and fundamental frequency $\omega_0 = 2\pi/T = \pi$, $y(t)$ is also periodic with period $T = 2$, and the Fourier series components of $y(t)$ are given by

$$Y_n = F_n H(\omega_0 n) = \frac{\cos(\pi n) + j\sin(\pi n)}{2n^2} = \frac{(-1)^n}{2n^2}$$

for all $n \neq 0$, and $Y_0 = F_0 H(0) = 1$. Thus we have

$$\begin{aligned}
y(t) &= \sum_{n=-\infty}^{\infty} Y_n e^{j\omega_0 nt} = 1 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(-1)^n}{2n^2} e^{j\pi nt} \\
&= 1 + \sum_{n=1}^{\infty} \frac{(-1)^{-n}}{2(-n)^2} e^{-j\pi nt} + \frac{(-1)^n}{2n^2} e^{j\pi nt} \\
&= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2} \left(e^{j\pi nt} + e^{-j\pi nt}\right) \\
&= 1 + \sum_{n=1}^{\infty} (-1)^n \frac{\cos(\pi nt)}{n^2} = 1 - \cos(\pi t) + \frac{1}{4}\cos(2\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{16}\cos(4\pi t) + \cdots
\end{aligned}$$

Problem 2.6

Find the Fourier series components F_n of $f(t) = \sin^4(t)$.

Solution:

We could use the standard method of integrating $f(t)e^{-j\omega_0 nt}$ in a period to find F_n; however, using Euler's formula, we have

$$\begin{aligned}
\sin^4(t) &= \left(\frac{e^{jt} - e^{-jt}}{2j}\right)^4 = \frac{1}{16}\left(\left(e^{jt} - e^{-jt}\right)^2\right)^2 \\
&= \frac{1}{16}\left(e^{2jt} + e^{-2jt} - 2\right)^2 \\
&= \frac{1}{16}\left(e^{4jt} + e^{-4jt} - 4e^{2jt} - 4e^{-2jt} + 6\right).
\end{aligned}$$

Thus $\omega_0 = 2$ and

$$F_n = \begin{cases} 1/16 & n = \pm 2 \\ -1/4 & n = \pm 1 \\ 3/8 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Problem 2.7

Determine the Fourier series coefficients F_n and find the average power in a period of the function:

$$\sum_{n=-\infty}^{\infty} x(t-4n) \qquad \text{where} \qquad x(t) = \begin{cases} 1 & 0 \le t < 2 \\ 2 & 2 \le t < 4 \end{cases}$$

Solution:

Note that for all t, we have $f(t) = f(t-4)$, so $f(t)$ is periodic with period $T = 4$, so $\omega_0 = 2\pi/T = \pi/2$. The average power in a period is:

$$\frac{1}{4}\int_0^4 f(t)^2\,dt = \frac{1}{4}\left(\int_0^2 1\,dt + \int_2^4 4\,dt\right) = \frac{9}{4}$$

To calculate F_n:

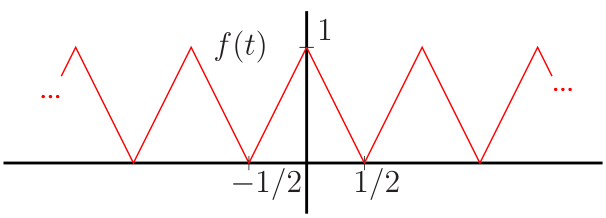
$$\begin{aligned}
F_n &= \frac{1}{T}\int_T f(t)e^{-j\omega_0 nt} \\
&= \frac{1}{4}\left(\int_0^2 e^{-j\pi nt/2}\,dt + \int_2^4 2e^{-j\pi nt/2}\,dt\right) \\
&= \frac{1}{4}\left(\frac{e^{-j\pi n}-1}{-j\pi n/2} + \frac{2e^{-j2\pi n}-2e^{-j\pi n}}{-j\pi n/2}\right) && \text{(Assume } n \neq 0\text{)} \\
&= \frac{j}{2\pi n}\left(e^{-j\pi n} - 1 + 2 - 2e^{-j\pi n}\right) && (e^{j2\pi n} = 1 \text{ for all integers } n) \\
&= \frac{j}{2\pi n}\left(1 - e^{-j\pi n}\right) \\
&= \begin{cases} j/\pi n & n \text{ odd} \\ 0 & n \text{ even and } n \neq 0 \end{cases} && (e^{j\pi n} = (-1)^n)
\end{aligned}$$

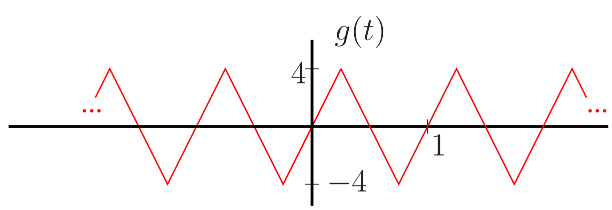
When $n = 0$, we divide by 0 in the third line, so we need to calculate F_0 separately

$$F_0 = \frac{1}{4}\int_0^4 f(t) = \frac{3}{2}.$$

Problem 2.8

Find the Fourier series components of $f(t)$ and $g(t)$ and write $f(t)$ and $g(t)$ as purely real sums of sine and/or cosine functions, where





Hint: After calculating the components of $f(t)$, use the properties to find the components of $g(t)$.

Solution:

We have $T = 1$, so $\omega_0 = 2\pi$, and in a period from $-1/2$ to $1/2$, $f(t) = 1 - |2t|$. So for all $n \neq 0$, we have

$$
\begin{aligned}
F_n &= \frac{1}{T}\int_{-1/2}^{1/2} f(t) e^{-jn\omega_0 t}\, dt \\
&= \int_{-1/2}^{0} (2t+1)\, e^{-j2\pi nt}\, dt + \int_{0}^{1/2} (1-2t)\, e^{-j2\pi nt}\, dt \\
&= \int_{-1/2}^{0} 2t\, e^{-j2\pi nt}\, dt - \int_{0}^{1/2} 2t\, e^{-j2\pi nt}\, dt + \int_{-1/2}^{1/2} e^{-j2\pi nt}\, dt \\
&= \frac{2j\pi nt + 1}{2(\pi n)^2}\, e^{-j2\pi nt}\Big|_{-1/2}^{0} - \frac{2j\pi nt + 1}{2(\pi n)^2}\, e^{-j2\pi nt}\Big|_{0}^{1/2} + \frac{e^{-j2\pi nt}}{-j2\pi n}\Big|_{-1/2}^{1/2} \\
&= \frac{(1 - (1 - j\pi n)\, e^{j\pi n}) - ((j\pi n + 1)e^{-j\pi n} - 1)}{2\pi^2 n^2} + \frac{e^{-j\pi n} - e^{j\pi n}}{-j2\pi n} \\
&= \frac{2 - (1 - j\pi n)(-1)^n - (1 + j\pi n)(-1)^n}{2\pi^2 n^2} \\
&= \frac{1 - (-1)^n}{\pi^2 n^2} \\
&= \begin{cases} 2/(\pi n)^2 & n \text{ is odd} \\ 0 & n \text{ is even and } n \neq 0 \end{cases}
\end{aligned}
$$

When $n = 0$, we divide by 0 in the fourth line, so we need to calculate F_0 separately

$$
\begin{aligned}
F_0 &= \frac{1}{T}\int_T f(t)\, dt \\
&= \int_{-1/2}^{0} (2t+1)\, dt + \int_{0}^{1/2} (1-2t)\, dt = \frac{1}{2}
\end{aligned}
$$

Thus

$$
\begin{aligned}
f(t) &= \frac{1}{2} + \sum_{n \text{ odd}} \frac{2e^{j2\pi nt}}{\pi^2 n^2} \\
&= \frac{1}{2} + \frac{2}{\pi^2}\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{e^{j2\pi nt} + e^{-j2\pi nt}}{n^2} \\
&= \frac{1}{2} + \frac{4}{\pi^2}\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\cos(2\pi nt)}{n^2}
\end{aligned}
$$

We have $g(t) = 8(f(t - 1/4) - 1/2) = 8f(t - 1/4) - 4$, so by the Amplitude scaling and time-shift properties of the Fourier series:

$$G_n = 8F_n e^{-j\pi n/2}$$

for all $n \neq 0$ and

$$G_0 = 8F_0 - 4 = 0$$

and so

$$G_n = \begin{cases} 0 & n \text{ is even} \\ 16(-j)^n/(\pi^2 n^2) & n \text{ is odd} \end{cases}$$

Also

$$\begin{aligned} g(t) &= 8f(t-1/4) - 4 \\ &= \frac{32}{\pi^2} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\cos(2\pi nt - \pi n/2)}{n^2} \end{aligned}$$

Problem 2.9

Find the Fourier series components of $f(t)$. Using the properties of the Fourier series, find the Fourier series coefficients of $g(t)$, where

$$f(t) = 2 + \sum_{n=-\infty}^{\infty} x(t-4n) \quad \text{and} \quad g(t) = \sum_{n=-\infty}^{\infty} y(t-4n)$$

$$x(t) = \begin{cases} t-2 & \text{when } 0 \le t < 4 \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \quad y(t) = \begin{cases} t^2/4 & \text{when } -2 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Hint: write $x(t)$ in terms of $y(t)$ and write $f(t)$ in terms of $g(t)$.

Solution:

In the period $[0, 4]$, $f(t) = t$, so for all $n \neq 0$, we have

$$\begin{aligned} F_n &= \frac{1}{T} \int_0^T t e^{-j\omega_0 nt} \, dt \\ &= \frac{j\omega_0 nt - 1}{T\omega_0^2 n^2} e^{-j\omega_0 nt} \Big|_0^T \\ &= \frac{j\omega_0 nT - 1}{T\omega_0^2 n^2} e^{-j\omega_0 nT} + \frac{1}{T\omega_0^2 n^2} \\ &= \frac{j2\pi n - 1}{2\pi\omega_0 n^2} e^{-j2\pi n} + \frac{1}{2\pi\omega_0 n^2} \\ &= \frac{j}{\omega_0 n} = \frac{2}{j\pi n} \end{aligned}$$

and

$$F_0 = \frac{1}{4} \int_0^4 t \, dt = 1.$$

Note that $x(t) = 2\frac{d}{dt}y(t-2)$, so we have $f(t) = 2 + 2\frac{d}{dt}g(t-2)$, and both $f(t)$ and $g(t)$ are periodic with period 4. Then by the amplitude-scaling, time-derivative, and time-shift properties, we have $F_n = 2\, j\omega_0 n\, G_n\, e^{-j2\omega_0 n}$, for all $n \neq 0$. Thus

$$G_n = F_n \frac{(-1)^n}{j\pi n} = \frac{2}{\pi^2 n^2} (-1)^{n+1}$$

and $F_0 = 2G_0 + 2$, so

$$G_0 = \frac{F_0 - 2}{2} = -\frac{1}{2}$$

Problem 2.10

Suppose $f(t)$ is the input to an LTI system with transfer function $H(\omega)$, where

$$f(t) = |\sin(\pi t)| \text{ and } H(\omega) = 1 - 4\left(\frac{\omega}{2\pi}\right)^2$$

Find the output $y(t)$ and write it as a purely real sum of sines and/or cosines.

Solution:

$|\sin(\pi t)|$ repeats itself twice as often as $\sin(\pi t)$, since $|\sin(\pi t)|$ is always positive. Thus the period of $|\sin(\pi t)|$ is 1 and $\omega_0 = 2\pi$.

In the period $[0, 1]$, $|\sin(\pi t)| = \sin(\pi t)$, so we can calculate the Fourier series components of $f(t)$ by

$$\begin{aligned}
F_n &= \frac{1}{T}\int_T |\sin(\pi t)|e^{-j\omega_0 nt}\,dt \\
&= \int_0^1 \sin(\pi t)\,e^{-j2\pi nt}\,dt \\
&= \frac{1}{2j}\int_0^1 (e^{j\pi t} - e^{-j\pi t})\,e^{-j2\pi nt}\,dt \\
&= \frac{1}{2j}\int_0^1 e^{j\pi t(1-2n)} - e^{-j\pi t(1+2n)}\,dt \\
&= \frac{1}{2j}\left(\frac{e^{j\pi(1-2n)} - 1}{j\pi(1-2n)} - \frac{e^{-j\pi(1+2n)} - 1}{-j\pi(1+2n)}\right) \\
&= \frac{-1}{2\pi}\left(\frac{-2}{1-2n} + \frac{-2}{1+2n}\right) \\
&= \frac{1}{\pi}\left(\frac{(1+2n)+(1-2n)}{(1-2n)(1+2n)}\right) \\
&= \frac{2}{\pi(1-4n^2)}
\end{aligned}$$

Since $f(t)$ is a sinusoidal function with period 1, $y(t)$ is also a sinusoidal function with period 1, where the Fourier series components given by

$$Y_n = F_n H(\omega_0 n) = \frac{2}{\pi(1-4n^2)}(1-4n^2) = \frac{2}{\pi}$$

Thus we have

$$\begin{aligned}
y(t) &= \sum_{n=-\infty}^{\infty} Y_n e^{j\omega_0 nt} \\
&= \frac{2}{\pi}\sum_{n=-\infty}^{\infty} e^{j2\pi nt} \\
&= \frac{2}{\pi}\left(1 + \sum_{n=1}^{\infty} e^{-j2\pi nt} + e^{j2\pi nt}\right) \\
&= \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\cos(2\pi nt)
\end{aligned}$$

MATLAB Problem 3

In this problem, I am providing you with four noisy vectors of length $N = 11613$. Embedded in one of these vectors is a famous quote from a former politician. Your goal will be to determine which of these four vectors contains the audio signal (and determine which three are purely noise) and decode the audio signal.

- Place the files 'one.mat', ..., 'four.mat' in your MATLAB directory.
- Run the commands 'load one.mat;', ... 'load four.mat;' to load the vectors.
- Declare variables '$F_s = 11025$; and '$t = (0 : N-1)/F_s$;'

  (F_s is the frequency the audio message was sampled at, and t is an array of time (in seconds) you can use to plot. The sampling frequency essentially tells us that all frequencies present in the signal are at most $\frac{F_s}{2}$. We will learn more about this later)
- Try plotting 'one', 'two', 'three', and 'four' and running 'sound(one, F_s);',..., 'sound(four, F_s);'

  (warning the sound will be unpleasant).

  You will likely be unable to determine which signal contains the message by examining these signals in the time domain.
- Fortunately for us, the noise I used only has frequencies outside of the range of frequencies in the audio clip. So while the signals are garbled in the time domain, we may be able to decipher some information in the frequency domain.
- Try plotting the magnitude of each signal in the frequency domain.

  To do this, run the following commands:

  '$f = (-F_s/2 : \; F_s/(N-1) \; : F_s/2)$;'

  (this creates a length-N array whose entries range from $\frac{-F_s}{2}$ to $\frac{F_s}{2}$ and increment by $\frac{F_s}{N-1}$)

  'One = fft(one);'

  (this creates an array of length N that represents one in the frequency domain)

  'plot(f, abs(fftshift(One)));'

  If they still all look similar to you, try plotting the logarithm of the absolute value of the magnitude of the signal in the frequency domain.

  'plot(f, $\log$(abs(fftshift(One))));'

  By doing this with all four signals, you should notice that one of the signals contains frequency components that the other three do not.
- Once you have identified the signal with the audio embedded in it, you will need to filter out the noise. To do this, we can use an ideal high, low, or band-pass filter, and we can work entirely in the frequency domain.

  Let 'X' denote the frequency domain array (i.e. $X = \text{fft}(x)$) of the signal with the audio embedded in it. I have provided you with a function for an ideal band-pass filter (HW2_Filter.m), which you can use for filtering by doing something like:

  '$Z = X \; .* $ HW2_Filter(f, A, B);'

  You will need to experiment with the values of A and B.

  Alternatively, you can figure out another way to filter out the noise, if you'd like.

- Finally, once you have filtered out the noise in the frequency domain, you need to convert Z back into the time domain. You can do this by taking:

  '$z = \text{real}(\text{ifft}(Z))$;'

  'sound(z, F_s);'

  If you implemented the filtering correctly, you should hear the audio.

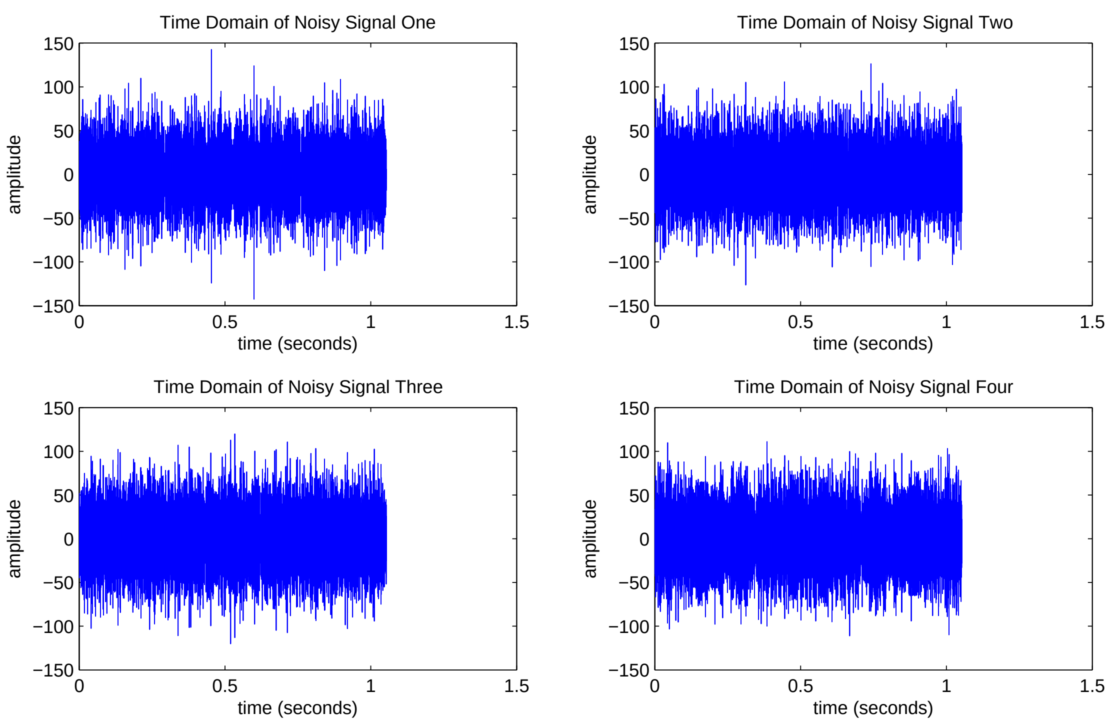
Submit the following:

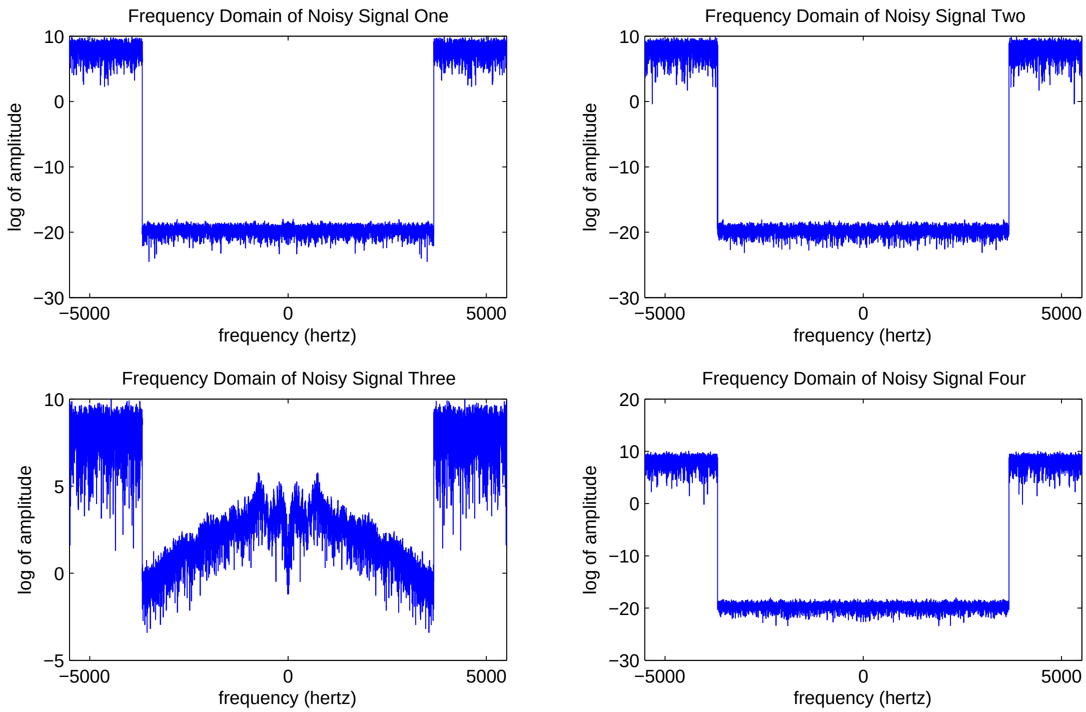
- which signal has the audio embedded in it
- the contents of audio message
- a plot of the magnitude frequency domain representation of the unfiltered signal (with proper labeling, of course), and a plot of the magnitude of the frequency domain representation of the filtered audio message.

Solutions

See 'ECE45_MATLAB2.m' for implementation details.

The message is "I'll be back" spoken by former California governator Arnold Schwarzenegger in the 1984 movie 'The Terminator.' The message was embedded in signal 3. Plots in the time and frequency domain are given below.





It can be seen that each signal has noise present in the frequencies in the range 3600 to $F_s/2$, but only signal 3 has a significant amount of frequency components in the range 0 to 3600. After applying a low-pass filter with cut-off frequency 3600 Hz, we obtain the following: