Book Review: Wave Theory of Information

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"Wave Theory of Information", Massimo Franceschetti (Cambridge University Press 2018; 451 pp) Reviewed by Thomas L. Marzetta.

Some years ago, I was privileged to hear a lecture by Prof. Massimo Franceschetti, *On Landau's Eigenvalue Theorem and Information Cut-Sets*, in the Bell Labs Henry Landau Seminar. On that basis alone, I anticipated a great deal from Prof. Franceschetti's new book, *Wave Theory of Information*. I am happy to report that my prior expectations were fully justified.

This is an ambitious and important book. It comes at a time when, once again, the wireless industry and the academic research community face the existential question, **Is the PHY Layer Dead?** [1]. Eight years ago, the emergence of Massive MIMO answered this question with a resounding **No!** But now Massive MIMO is a commercial reality, and together with Millimeter Wave it dominates 5G wireless technology. Should a new physical layer breakthrough fail to emerge, then all future development will be limited to a mere adaptation of existing principles to shorter wavelengths.

One possible avenue to a breakthrough would be a unification of wireless communication theory and electromagnetic theory, something that has never been achieved [2]. It is (or should be) an embarrassing fact that all of today's wireless systems rely on exceedingly elementary models for the function of antennas and the propagation of signals, and Maxwell's equations play only a peripheral role within the theory that underlies these systems. A genuine fusion of electromagnetic theory with communication theory presents fascinating new research problems, and the incorporation of neglected physics in the formulation of new theories has a chance of yielding breakthroughs. Unfortunately, researchers in electromagnetics and in communication theory have shown little inclination to learn each other's field. As the Preface makes clear, Prof. Franceschetti aims to break down the barriers between these fields.

Chapter 1 introduces a recurring theme of degrees of freedom of a communication system. A bandlimited temporal signal has essentially $2WT$ degrees of freedom. Analogously, wave propagation physics implies that a closed region of space has an essential number of degrees of freedom that is proportional to the *area* of the surface of the region, measured in units of wavelength-squared, and not the volume. Together, the temporal and spatial dimensions provide a total number of degrees of freedom, that may be regarded as the number of parallel channels that comprise the link. In turn, the noisiness of the receivers dictates the precision with which signals can be distinguished, and, in combination with the number of degrees of freedom, determines the capacity of the system.

Chapter 2 treats bandlimited temporal signals from two points of view: the sampling theorem (the *Folk Theorem*), and more precisely, prolate spheroidal wave functions. For one who learned spheroidal wave functions from Van Trees [3], Prof. Franceschetti's presentation is illuminating: these functions are, in fact, solutions to the one-dimensional Helmholtz equation (the temporal Fourier transform of the wave equation) when expressed in a particular coordinate system!

Chapter 3 introduces the Hilbert-Schmidt integral operator, which is the continuous time (more generally, space/time) analog to the familiar singular value decomposition, in which the left and right eigenvectors are replaced by a countably-infinite number of left and right orthogonal eigenfunctions. Comparatively unknown among engineers, the Hilbert-Schmidt decomposition is the natural tool for formulating continuous-time linear least-squares problems [4].

Chapter 4 is a classical treatment of Maxwell's equations: the field is represented by a vector-valued potential, and a scalar potential, the substitution of which into Maxwell's equations implies that each of the four potentials satisfies the scalar wave equation. The complete solution, in space/time coordinates, is presented for an electric dipole source. The absence of temporal frequency in the formulation motivates an unconventional, wide-band definition of the Fraunhofer distance for a line-source of length L: $(L/\sqrt{2})$, instead of the usual definition $L^2/2\lambda$ (the range beyond which the curvature of an incoming spherical wave can be neglected).

Chapter 5 treats Maxwell's equations in terms of an equivalent linear space/time system. The Green's function—the space/time impulse response—is the natural tool for describing the solution.

Chapter 6 discusses non-line-of-sight propagation, which, when dense multipath scattering is involved, is best treated stochastically. The Green's function becomes a non-stationary random field, for which the Karhunen-Loeve decomposition provides the ideal representation.

Chapter 7 reviews some existing wireless communication techniques and typical problems encountered in wireless system design: inter-symbol interference, multiplexing, and diversity.

Chapters 8–10 bring all the previous material together, resulting in a remarkably simple description of propagation between a transmitting region of space and a receiving region of space in terms of the space/time Hilbert-Schmidt decomposition. The number of degrees of freedom is simply the number of significant singular values, which for temporally wide-band signals is obtained by calculating the degrees of freedom at each resolvable frequency, and then integrating over the signal bandwidth. The number of degrees of freedom for the system can be no greater than the minimum of the number of degrees of freedom associated with the transmitter and receiver regions. This is analogous to the classical point-to-point MIMO result that the number of degrees of freedom is no greater than the minimum of the number of transmit and receive antennas.

Chapter 11 is all about noise processes, and it includes classical derivations of black-body radiation, and Johnson noise in resistors. The treatment of noise for wireless receivers could have been improved by including an explicit list of sources of noise (external natural thermal noise, external man-made noise, Johnson noise arising from imperfectly conducting antenna components, and noise internal to the pre-amplifier). A beginner might be tempted to assume, incorrectly, that the antenna itself is a source of noise, e.g., by gratuitously substituting the radiation resistance of the antenna into the Johnson noise formula. In fact, a lossless antenna (where all conducting components have zero-resistivity) itself contributes nothing to receiver noise.

Chapter 12 is an exposition of Shannon information theory, via sphere packing, and also random coding. The chapter also presents Kolmogorov's notion of capacity in a deterministic setting. It would have been useful to point out that, while random codes are only decodable by exhaustive search, practical codes, such as turbo or LDPC, can operate very close to capacity.

Chapter 13 deals with universal entropy bounds. It is gratifying to read that the data storage capacity of a typical flash memory card could theoretically be improved by a factor as great as 10^{26} !

The goal of the book is more ambitious than merely educating electromagnetic theorists about Shannon theory, and educating communication theorists about electromagnetic theory. Throughout the book there are forays into diverse topics, such as Heisenberg's Uncertainty Principle, blind sensing, compressed sensing, communication network strategies, quantum mechanical corrections to the classical black body radiation and Johnson noise theories, gravitational limits (e.g., black holes), and rate-distortion theory. The reader can easily skip around these peripheral topics, if desired.

The book is exceedingly well written, and surprisingly thin, given the amount of material. The mathematics, supplemented by considerable intuitive explanation, is never overwhelming, and should be readily followed by the diligent reader. There are extensive references, and a useful summary at the end of each chapter, along with well-crafted exercises.

Unquestionably this book will contribute hugely to Prof. Franceschetti's goal: "to break through the compartmentalized walls of several disciplines". That said, there are some aspects of Shannon theory and electromagnetic theory which might have been emphasized to a greater extent, with good effect.

The book emphasizes connections between Shannon theory and classical thermodynamics; for example, Shannon entropy and thermodynamic entropy. But it is important to keep in mind that Shannon theory is completely self-consistent as an axiomatic mathematical theory, involving nondimensional quantities only. Valid physical interpretations of Shannon theory do not emerge automatically. It is all too easy to fall into the trap of assigning a physical interpretation merely because of similarity of terminology. The quantity E_b in the celebrated formula $(E_b/N_0) > \ln 2$, does not necessarily represent physical energy extracted from an antenna!

For a communication theorist, electromagnetic theory can be intimidating. Yet, looking beyond the mathematical complications, there are some results of supreme generality, but supreme simplicity. No matter what takes place in a closed volume of space, the resulting external electromagnetic field can be exactly represented as a linear combination of plane-waves, both ordinary and evanescent [5]. (This is an extension of the Weyl plane-wave representation for the spherical wave [6].) Point-to-point MIMO in an arbitrary scattering environment can be rigorously formulated in terms of the generation of plane-waves, the scattering of plane-waves, and the reception of plane-waves [7]. The interactions among any system of antennas, operating in a linear time-invariant medium (including passive scattering and dynamic mutual coupling) is rigorously described by old-fashioned Ernst Guillemin network theory: a complex-valued impedance matrix as a function of temporal frequency. For a real communication system, the computation of the plane-wave amplitudes and the mutual impedances requires elaborate numerical techniques, but these details should be of little concern to the communication theorist: the mere existence of the plane-wave representation and the impedance matrix provides the essential physics for the formulation of a physics-based communication theory.

To summarize: any researcher who purports to work on the advancement of wireless communication theory should take time to study *Wave Theory of Information*. Prof. Franceschetti's message, over-all, is profoundly optimistic: "Engineers are far from reaching the limits that nature imposes on communication: our students have a bright future in front of them!"

References

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