

The Capacity of Wireless Networks: Information-Theoretic and Physical Limits

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Abstract—It is shown that the capacity scaling of wireless networks is subject to a fundamental limitation which is independent of power attenuation and fading models. It is a degrees of freedom limitation which is due to the laws of physics. By distributing uniformly an order of n users wishing to establish pairwise independent communications at fixed wavelength inside a two-dimensional domain of size of the order of n , there are an order of n communication requests originating from the central half of the domain to its outer half. Physics dictates that the number of independent information channels across these two regions is only of the order of \sqrt{n} , so the per-user information capacity must follow an inverse square-root of n law. This result shows that information-theoretic limits of wireless communication problems can be rigorously obtained without relying on stochastic fading channel models, but studying their physical geometric structure.

Index Terms—Ad hoc networks, capacity, network information theory, scaling laws, wireless networks.

I. INTRODUCTION

A NATURAL question that arises is whether information theory can provide fundamental bounds on the capacity of wireless *ad hoc networks*, which are not tied to *ad hoc physical channel models*. One aim of this paper is to show that this is indeed the case, if the information-theoretic approach is appropriately combined with the study of the physics of wave propagation. The main contribution, however, should be seen in a broader context. Relying on functional analysis to study the vector space of the propagating field, rather than assuming stochastic fading channel models, could be a rigorous way to tackle other wireless communication problems.

The information theoretic characterization of the capacity region of wireless networks is one of the holy grails in information theory. It is a problem of great mathematical depth and

engineering interest. One way to approach the problem is due to Gupta and Kumar [14], who proposed to study the simpler case in which all the nodes in the network are required to transmit at the same bit-rate, and to look at the *scaling limit* of the achievable rate, as the number of nodes in the network grows. In this way, the capacity region collapses to a single point and order results on its behavior are obtained. Gupta and Kumar's bounds were also derived under some additional assumptions on the physics of propagation, and on some restrictions on the communication strategy employed by the nodes (i.e., multihop operation and pairwise coding and decoding). Later, starting with the work of Xie and Kumar [39], *information-theoretic* scaling laws, independent of any strategy used for communication, have been established by many authors. These results, however, heavily depend on the assumptions made on the electromagnetic propagation process. Presence or absence of fading, choice of fading models, and choice of path loss models, lead to different lower and upper bounds on the scaling limit of the information rate. As a consequence, a plethora of articles appeared in the information-theoretic literature [2], [3], [9], [12], [17], [19], [20], [24]–[28], [40], [41], presenting bounds ranging from a per-node rate that rapidly decays to zero as the number of nodes in the network tends to infinity, to bounds predicting a slower decay, to bounds that are practically constant. In these works, while the lower bounds rely on different cooperative schemes employed by the nodes, the upper bounds follow from the application of the same mathematical tool: the information-theoretic cut-set bound [6, Ch. 15]. This single strategy of attack, and the resulting dependence on ad hoc physical propagation models, are somehow undesirable for a theory that seeks the fundamental limits of communication.

In the same two-dimensional geometric setting of the works above, this paper shows that there exists a single scaling law, which is essentially an inverse-square-root of n law, and is dictated by Maxwell's physics of wave propagation, in conjunction to a Shannon-type cut-set bound. The result is then generalized to a three-dimensional setting at the end of the paper. The main contribution in 2-D is expressed as follows.

Claim: In a wireless network composed of n uniformly distributed nodes subject to an individual (or total) power constraint, operating at a fixed wavelength inside a two-dimensional domain of area n (normalized to the wavelength), and which are uniformly paired into sources and destinations, each source can communicate to its intended destination at most at rate $O((\log n)^2/\sqrt{n})$ bits per second. This scaling law is due to a limitation of the spatial degrees of freedom of the network that is independent of empirical path-loss models and

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stochastic fading models, but depends only on the geometric two-dimensional configuration of the network.

By looking closely at the claim above, we see a reflection of what Shannon has showed us, namely that the information capacity is limited by the power available for communication, *and* by the diversity available in the physical channel. In classical information theory, this diversity is expressed in terms of available frequency bandwidth. In the case of spatially distributed systems, such as wireless networks, this diversity constraint also appears in space. The usual approach of postulating stochastic fading channel models hides the explicit computation of the spatial diversity, while our analysis reveals it.

Being aware of such a fundamental limitation is certainly desirable, but what conclusions can be drawn from it on the optimal design and operation of wireless networks? Unfortunately not many. As it is often the case with fundamental limits, their generality can also be the curse of their practical applicability. But we are not left completely empty handed of engineering guidelines either. One important implication is that any cooperative communication scheme cannot achieve a rate higher than what is stated in the claim above, at least in the scaling limit sense. Physics simply forbids it. Mathematical proofs of higher capacity scaling [2], [12], [24]–[26], [28], achieved using sophisticated cooperative communication schemes, rely on stochastic channel models and in a strict scaling limit sense are artifacts of such models. This highlights the importance of using appropriate mathematical models of reality to derive information-theoretic results. But does this also lead to the irrefutable conclusion that sophisticated cooperative strategies such as network coding, space–time coding, hierarchical cooperation, do not lead to any gain? The general answer is no. Scaling results are only up to order and pre-constants can make a huge difference in practice. Sophisticated cooperative communication schemes could in principle be extremely beneficial in networks of any fixed size. A rigorous proof of this latter statement is, however, difficult to obtain in a non-limiting scenario, and should take into account many practical issues related to protocol overhead, like decentralized medium access synchronization, and availability of channel state information.

Finally, we wish to spend some additional words on the mathematical techniques we use in this paper. Resolving the amount of information that can be communicated through wave propagation is a venerable subject that has been treated by a great number of authors in different fields. Papers in optics often refer to the early works of Toraldo di Francia [37], [38]. In the mid nineteen-eighties the problem has been considered again in a more general context by Bucci and Franceschetti [4], [5], who introduced the important concepts of spatial bandwidth and degrees of freedom of scattered fields, and placed them into a rigorous functional analysis framework. Their results have been later extended to more complex geometries by Bucci, Gennarelli, and Savarese [34]. More recently, the problem has been treated by the works of Miller [23], Piestum and Miller [32], Poon, Brodersen and Tse [33], and Migliore [22]. Our mathematical framework follows the approach of Bucci and Franceschetti, which we find to be the most rigorous, and does not require far-field approximations. There are some important differences, however. Bucci and Franceschetti first establish the spatial bandlimitation property of the field in their first paper [4], and then they consider the problem of field reconstruction

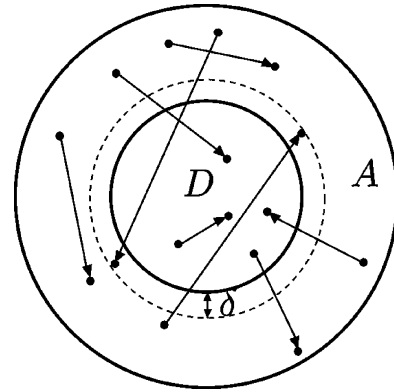


Fig. 1. The partition considered in the analysis.

from a bounded observation set in their second paper [5], using prolate spheroidal functions, which are known to be optimal in the Landau–Pollack–Slepian sense. Given the specifics of our problem, we do not need this full machinery, but only inherit its main philosophy. We follow a singular value decomposition approach, which is standard in communication theory, and use simpler basis functions for the field expansion, which are good enough for our purposes. We then look directly at the behavior of the singular values of this decomposition, without performing a space-band transform. This leads to simpler computations and shortens the treatment considerably.

The next section formally defines the problem and outlines its solution. Section III completes the solution by studying the dimension of the Hilbert space spanned by the electromagnetic vector field. Section IV presents the extension to a three-dimensional geometry and a final discussion of the results is presented in Section V.

II. INFORMATION-THEORETIC APPROACH

Throughout the paper, we consider distance lengths normalized to the carrier wavelength λ . Consider a Poisson point process \mathcal{P} of unit density inside a disc \mathcal{D}_n of radius $\sqrt{2n}$, and partition \mathcal{D}_n into two equal parts by drawing a circular cut of radius \sqrt{n} at the origin, which divides \mathcal{D}_n into the inner disc D and the outer annulus A , where for convenience of notation we do not explicitly indicate the dependence on n . The points of the process represent the nodes of the network and we assume a uniform traffic pattern: nodes are paired independently and uniformly, so that there are an order of n communication requests that need to cross the boundary of the partition, see Fig. 1.

Assuming that each node in \mathcal{D}_n generates at most P watts,¹ we want to find an upper bound on the per-node communication rate $R(n)$ that all nodes can achieve simultaneously to their intended destinations. To do so, we consider the sum C_n of the rates that can be sent from the transmitters in D to the receivers in A . We have, with high probability (w.h.p.)

$$R(n) = O\left(\frac{C_n}{n}\right). \quad (1)$$

Next, to upper bound C_n we assume that the nodes on one side of the cut can share information instantaneously among themselves, and can also distribute the power among themselves in

¹Assuming a total power constraint rather than a per-node one does not change the results.

order to establish communication in the most efficient way with the nodes on the other side; which in turn are able to distribute the received information instantaneously among themselves. In this way, C_n is upper bounded by the capacity of a single user multiple-input multiple-output (MIMO) antenna array communicating across the partition.

The MIMO channel model across the cut is the space-vectorial version of the additive white Gaussian noise channel. In discrete time steps, it has the following representation:

$$Y_d[i] = \sum_{s \in \mathcal{P} \cap D} h_{sd}[i] X_s[i] + Z_d[i], \quad \text{for all } d \in \mathcal{P} \cap A \quad (2)$$

where $X_s[i]$ are the symbols sent by node s at time i , $Y_d[i]$ are the symbols received by node d at time i , and $Z_d[i]$ are (independent space-time) Gaussian variables with unit variance. The coefficients $h_{sd}[i]$ model the strength of the propagation channel between node s and node d and, given the realization of \mathcal{P} , are deterministically dictated by the physics through Maxwell equations. Throughout the paper we assume a fixed environment, i.e., $h_{sd}[i] = h_{sd}$ for all i , but it will be clear that our results do not change in a dynamic environment where the coefficients h_{sd} vary over time. In matrix form, (2) is rewritten as

$$\mathbf{Y}_A[i] = \mathbf{H} \mathbf{X}_D[i] + \mathbf{Z}_A[i]. \quad (3)$$

Considering coding across time using blocks of m symbols and denoting the mutual information between space-time codewords \mathbf{X}_D^m and \mathbf{Y}_A^m as $\mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_A^m)$, the information flow through the cut can be upper bounded as follows:

$$mC_n \leq \max_{p(\mathbf{X}_D^m)} \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_A^m). \quad (4)$$

$$\sum_{i=1}^m X_s^2[i] \leq mP, \forall s \in \mathcal{P} \cap D$$

We now divide the information flow across the cut into two contributions. Let V be the annulus of constant width $\delta > 0$ around D . The first contribution is the information flow from the nodes in D to the nodes in V . The second contribution is the information flow from the nodes in D to the nodes in $A \setminus V$. Formally

$$\begin{aligned} \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_A^m) &= \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_V^m, \mathbf{Y}_{A \setminus V}^m) \\ &\leq \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_V^m) + \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_{A \setminus V}^m) \end{aligned} \quad (5)$$

where the inequality holds as the space components of $\mathbf{Z}_A[i]$ are independent. Combining (4) and (5), it follows that

$$\begin{aligned} mC_n &\leq \max_{p(\mathbf{X}_D^m)} \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_V^m) \\ &\quad + \max_{p(\mathbf{X}_D^m)} \mathcal{I}(\mathbf{X}_D^m; \mathbf{Y}_{A \setminus V}^m) \\ &=: mC^{(V)} + mC^{(A \setminus V)}. \end{aligned} \quad (6)$$

$$\sum_{i=1}^m X_s^2[i] \leq mP, \forall s \in \mathcal{P} \cap D$$

Next, we consider the two terms in (6) separately and derive corresponding upper bounds. The first bound is obtained using standard information-theoretic arguments and relies only on the power constraint and on counting the number of transmitters and

receivers, while the second bound is obtained by merging the information theory with a more detailed physical analysis of the wave propagation process.

Let us start with the easy part: we bound $C^{(V)}$ by summing the capacities of the individual multiple-input single-output (MISO) channels between all nodes in D and each receiver in V . We have, w.h.p.

$$\begin{aligned} C_V &\leq \sum_{d \in \mathcal{P} \cap V} \frac{1}{2} \log \left(1 + \frac{P}{\sigma^2} \sum_{s \in \mathcal{P} \cap D} |h_{sd}|^2 \right) \\ &\leq K_1 \sqrt{n} \log \left(1 + \frac{P}{\sigma^2} K_2 n \max_{s \in \mathcal{P} \cap D, d \in \mathcal{P} \cap V} |h_{sd}|^2 \right) \\ &= O(\sqrt{n} \log n) \end{aligned} \quad (7)$$

where K_1, K_2 are positive constants. The first inequality is a standard information-theoretic cut-set bound. The second inequality is due to the number of nodes in V being w.h.p. $O(\sqrt{n})$ and the number of nodes in $A \setminus V$ being w.h.p. $O(n)$. The last equality is due to $\max_{s \in \mathcal{P} \cap D, d \in \mathcal{P} \cap V} |h_{sd}|^2 = O(n)$, as one can at most beamform the total transmitted power on a single channel. Physically, the bound in (7) shows something very simple: there are at most a constant times \sqrt{n} independent output channels, and the capacity of each of them is at most proportional to $\log n$, since the total transmitted power is of the order of n . Hence, the bound in this case is independent of the number of degrees of freedom that are effectively available in the physical channel and depends only on the total number of transmitting and receiving antennas.

We now focus on $C_{A \setminus V}$. In this case the number of degrees of freedom effectively available in the physical channel, rather than the total number of antennas available for communication, is the bottleneck that provides the required bound. To show this, we study the physics of the wave propagation process. We start by noting that $C_{A \setminus V}$ is independent of the nodes in V , so their presence does not increase the information flow and the upper bound can be computed assuming V to be empty. Thanks to the empty separation annulus V , the kernel of the propagation operator connecting transmitters and receivers does not have singularities due to receivers being arbitrarily close to transmitters, and we can study the degrees of freedom of such operator using a functional analysis approach. The result, formally derived in the next sections, is the following. Let $\mathbf{H}^{(A \setminus V)}$ be the matrix with entries $h_{sd}, s \in \mathcal{P} \cap D, d \in \mathcal{P} \cap (A \setminus V)$. Although $O(n)$ antennas are available in $A \setminus V$

$$\text{rank}(\mathbf{H}^{(A \setminus V)}) = O(\sqrt{n} \log n). \quad (8)$$

It then follows, by performing the same steps leading to (7) but summing only over the effective number of independent MISO channels given by (8), rather than over all the receiving nodes, that w.h.p.

$$C_{A \setminus V} = O(\sqrt{n} (\log n)^2). \quad (9)$$

Combining (6), (7), and (9), we have, w.h.p.,

$$C_n = O(\sqrt{n} (\log n)^2).$$

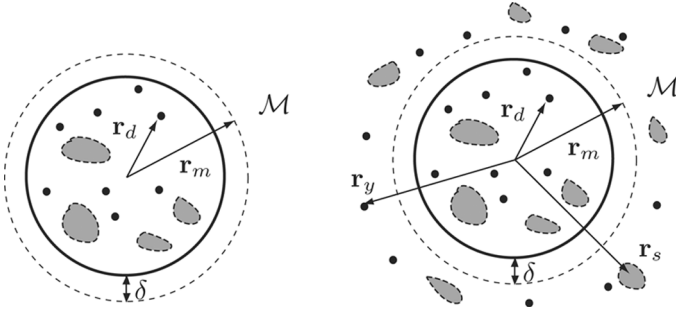


Fig. 2. Left-hand side: step one, free space propagation. Right-hand side: step two, propagation with scattering elements. Transmitting and receiving antennas are denoted by black dots, while scatterers are denoted in gray.

The final result now follows immediately from (1): w.h.p.,

$$R(n) = O\left(\frac{(\log n)^2}{\sqrt{n}}\right). \quad (10)$$

We make the following remark. The geometric setting considered above is by now standard in the literature, but it is not the most general one for which our result holds. We could have considered any arbitrary distribution of nodes in the disc D and in the annulus A and any matching between the nodes in the two regions. The only constraint on the distribution of the nodes is either that the node closest to the boundary of the partition must be at fixed distance δ from it, or that the number of nodes violating this minimum distance constraint is at most of $O(\sqrt{n})$, so that their contribution to the information flow can be bounded by a power constraint argument as in (7) rather than by a degrees of freedom argument.

III. THE PHYSICS OF THE INFORMATION FLOW

All that remains to be done is to provide a formal proof of (8). We do this in three steps. In a first step, we study the properties of the electromagnetic field that propagates up to distance $\delta > 0$ from the inner disc and is incident on the circumference \mathcal{M} , see the left-hand side of Fig. 2. In doing so, we assume to have an arbitrary collection of sources and scatterers placed inside the disc D , while the outside space is empty. Under these assumptions we show that the field incident on \mathcal{M} is completely described by a linear combination of $O(\sqrt{n} \log n)$ basis functions. In other words, the number of degrees of freedom of the incident field is $O(\sqrt{n} \log n)$. In a second step, we consider the presence of scatterers outside the circle \mathcal{M} and show that these do not change the number of degrees of freedom of the field incident on \mathcal{M} , see the right-hand side of Fig. 2. The intuitive justification of this latter fact is that the field backscattered on \mathcal{M} does not provide *new* information, since this has already passed through \mathcal{M} . Furthermore, we argue by the uniqueness theorem [15, p. 100] that the field at any point outside \mathcal{M} , and in particular at the receiving antennas, is given by a linear transformation of the field on \mathcal{M} , which does not change the number of degrees of freedom. Finally, in a third step, we notice that receiving antennas detect a voltage proportional to the intensity of the field incident on them, plus some thermal noise, and this leads to the desired information-theoretic result.

The physical channel model is summarized in Fig. 3, which shows the input-output relationship between transmitted and received signals. Such relationship is given by a chain of operators and corresponds to the information-theoretic channel model in (3). These operators are formally studied in the next sections according to the outline provided above. The figure shows that arbitrary source symbols represented by the input vector \mathbf{X}_D are mapped into a physical current density $I(\mathbf{r}_d)$ inside the disc D through the operator \mathcal{F} . Next, the currents in D are related to the field $E(\mathbf{r}_m)$ on \mathcal{M} through the free-space radiation operator \mathcal{G} . The operator \mathcal{D} accounts for the presence of scatterers outside \mathcal{M} and represents the mapping from the field on \mathcal{M} to the vector \mathbf{O} of the intensities of the electric field on the receiving antennas. Finally, the voltage at each receiving antenna is proportional to the intensity of the electric field incident on it, and the output symbol vector \mathbf{Y}_{AY} is given by the voltage on the antennas, plus some additive noise.

The proof outline described above can now be revisited in terms of the physical channel model depicted in Fig. 3. We show that the range space of the operator \mathcal{G} is of dimension $O(\sqrt{n} \log n)$, as n tends to infinity, and that the operator \mathcal{D} is linear and thus does not increase the dimension of the space. Similarly, the linear map $\mathbf{V} = K\mathbf{O}$ does not increase the dimension of the space. The range-space of \mathcal{F} can be assumed of arbitrary dimension, and we conclude that the range-space of $\mathbf{H}^{(A \setminus V)}$ is of dimension $O(\sqrt{n} \log n)$, as n tends to infinity.

A. Step One: Propagation in Free Space

In this section, we show that the electric field at any point on \mathcal{M} lies on a Hilbert space of dimension $O(\sqrt{n} \log n)$, as n tends to infinity. We assume sources and scatterers to be present in D , while the outside space is empty.

Being interested in an upper bound on the information flow, we can assume that the sources are arbitrarily located in D and can control the current density inside the disc D . We let such *arbitrary* current density be $I(\mathbf{r}_d)[\text{A}/\text{m}^2]$, $\mathbf{r}_d \in D$. Notice that singular sources can be thought of as limiting cases of two-dimensional distributions.

Assuming two-dimensional cylindrical propagation, so that the current density is \hat{z} directed, the electric field radiated by currents in D and observed at $\mathbf{r}_m \in \mathcal{M}$ has only the \hat{z} component, and is given by [15, pp. 223–232]

$$E(\mathbf{r}_m) = \frac{-\beta^2}{4\omega\epsilon_0} \int_D I(\mathbf{r}_d) H_0^{(2)}(\beta|\mathbf{r}_m - \mathbf{r}_d|) ds \quad \mathbf{r}_m \in \mathcal{M} \quad (11)$$

where ds is an element of area perpendicular to \hat{z} , $\beta = \frac{2\pi}{\lambda}$ is the wavenumber, ϵ_0 is the permittivity of the vacuum, $H_i^{(2)}(x)$ is the Hankel function of the second kind and order i , and a Fourier transform convention $\exp(j\omega t)$ has been adopted, $\omega = 2\pi/(\sqrt{\epsilon_0\mu_0}\lambda)$ being the angular frequency, and μ_0 being the permeability of the vacuum. Furthermore, we assume the following power constraint:

$$a \int_D |I(\mathbf{r}_d)|^2 ds \leq nP \quad (12)$$

wherein a is a normalization constant and P is the individual power constraint of each source. This condition ensures that the

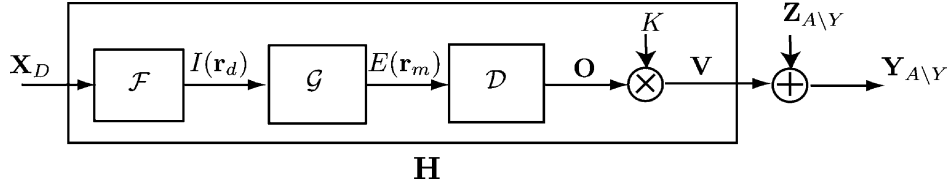


Fig. 3. The physical channel model.

power radiated by the sources is finite and linearly proportional to the number of sources in D . Equation (11) shows that the currents in D and the electric field on \mathcal{M} are linearly related through the radiation operator \mathcal{G} (whose kernel is the Green's function). It follows that (11) can be written as

$$E(\mathbf{r}_m) = (\mathcal{G}I)(\mathbf{r}_m), \quad \mathbf{r}_m \in \mathcal{M} \quad (13)$$

where

$$(\mathcal{G}I)(\mathbf{r}_m) = \frac{-\beta^2}{4\omega\epsilon_0} \int_D I(\mathbf{r}_d) H_0^{(2)}(\beta|\mathbf{r}_m - \mathbf{r}_d|) ds \quad (14)$$

represents the radiation operator, which maps a current density in D into the electric field at a point $\mathbf{r}_m \in \mathcal{M}$. We can also see from (14) the reason why we have introduced a minimum separation $\delta > 0$ between the sources and the observation domain \mathcal{M} , as the kernel of the radiation operator is singular at $\mathbf{r}_m = \mathbf{r}_d$. By introducing a separation $\delta > 0$ between the sources and the observation domain we avoid singularities in the kernel of (14), obtaining a compact integral operator with analytic kernel.

Next, we study the analytical properties of the operator \mathcal{G} , and show that the range-space of such operator is practically finite when the dimension of the radiating system is large. In order to do so, we represent the electric field on \mathcal{M} in terms of the Hilbert–Schmidt decomposition, that is the equivalent of the singular value decomposition for operators in the L^2 space, and show that the electric field in (11) is completely described by $O(\sqrt{n} \log n)$ singular functions, as $n \rightarrow \infty$. The Hilbert–Schmidt decomposition of (11) is given by

$$E(\mathbf{r}_m) = \sum_{k=-\infty}^{\infty} \sigma_k \langle I, v_k \rangle_{L^2} u_k(\mathbf{r}_m), \quad \mathbf{r}_m \in \mathcal{M} \quad (15)$$

where $\{\sigma_k\}$ are the singular values of the operator; u_k and v_k are the k -th left singular function and right singular function, respectively, and $\langle a, b \rangle_{L^2} := \int a(\mathbf{r}) b^*(\mathbf{r}) d\mathbf{r}$ denotes the inner product between functions in L^2 . In order to compute the singular values $\{\sigma_k\}$, it is convenient to choose the following set of orthonormal functions:

$$u_k(\mathbf{r}_m) = -\frac{H_k^{(2)}(2\pi(\sqrt{n} + \delta)) e^{jk\angle \mathbf{r}_m}}{\sqrt{2\pi}(\sqrt{n} + \delta)^{1/2} |H_k^{(2)}(2\pi(\sqrt{n} + \delta))|} \quad (16)$$

where $\mathbf{r}_m \in \mathcal{M}$, and

$$v_k(\mathbf{r}_d) = \frac{J_k(\beta|\mathbf{r}_d|) e^{jk\angle \mathbf{r}_d}}{\sqrt{2\pi} \left(\int_0^{\lambda\sqrt{n}} |J_k(\beta r_d)|^2 r_d dr_d \right)^{1/2}} \quad (17)$$

where $\mathbf{r}_d \in D$, $J_k(x)$ is the Bessel function of the first kind and order k , and $|\mathbf{r}|$ and $\angle \mathbf{r}$ are the magnitude and the angular coordinate of the vector \mathbf{r} respectively. Using the addition theorem for Hankel functions [15, p. 232], we can write $H_0^{(2)}(\beta|\mathbf{r}_m - \mathbf{r}_d|)$ in terms of cylindrical wave functions referred to the origin, and (11) can be rewritten as

$$E(\mathbf{r}_m) = \frac{-\beta^2}{4\omega\epsilon_0} \int_D I(\mathbf{r}_d) \sum_{k=-\infty}^{\infty} J_k(\beta|\mathbf{r}_d|) \times H_k^{(2)}(\beta|\mathbf{r}_m|) e^{jk\angle(\mathbf{r}_m - \mathbf{r}_d)} ds. \quad (18)$$

Comparing (15) and (18), and using (16) and (17), we immediately obtain that

$$\sigma_k = \frac{\pi\beta^2}{2\omega\epsilon_0} \left| H_k^{(2)}(2\pi(\sqrt{n} + \delta)) \right| (\sqrt{n} + \delta)^{1/2} \times \left(\int_0^{\lambda\sqrt{n}} |J_k(\beta r_d)|^2 r_d dr_d \right)^{1/2}. \quad (19)$$

The integral in (19) can be solved using identity (5.54.2) in [13], yielding

$$\int_0^{\lambda\sqrt{n}} |J_k(\beta r_d)|^2 r_d dr_d = \frac{x^2}{2} \left[(J_k(\beta x))^2 - J_{k-1}(\beta x) J_{k+1}(\beta x) \right] \Big|_0^{\lambda\sqrt{n}}. \quad (20)$$

Substituting (20) into (19) we obtain the following expression for the singular values of the operator \mathcal{G} :

$$\sigma_k = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\sqrt{2}}{2} \pi^2 \sqrt{n} (\sqrt{n} + \delta)^{1/2} \left| H_k^{(2)}(2\pi(\sqrt{n} + \delta)) \right| \times \left((J_k(2\pi\sqrt{n}))^2 - J_{k-1}(2\pi\sqrt{n}) J_{k+1}(2\pi\sqrt{n}) \right)^{1/2}. \quad (21)$$

The electric field incident on \mathcal{M} lies on a Hilbert space whose dimension depends on the behavior of the singular values in (21) as a function of the index k . It turns out that these are approximately constant up to a *critical value* $k_c \approx 2\pi\sqrt{n}$, after which they undergo a phase transition and rapidly decay to zero. The transition tends to become a step function as $n \rightarrow \infty$, as shown in Fig. 4. This leads to the conclusion that the electric field can be represented with a vanishing error using roughly k_c basis functions. This latter claim is made precise in the next theorem, proven in the Appendix.

Theorem 3.1: Let

$$\hat{E}_N(\mathbf{r}_m) = \sum_{k=-N}^N \sigma_k \langle I, v_k \rangle_{L^2} u_k(\mathbf{r}_m).$$

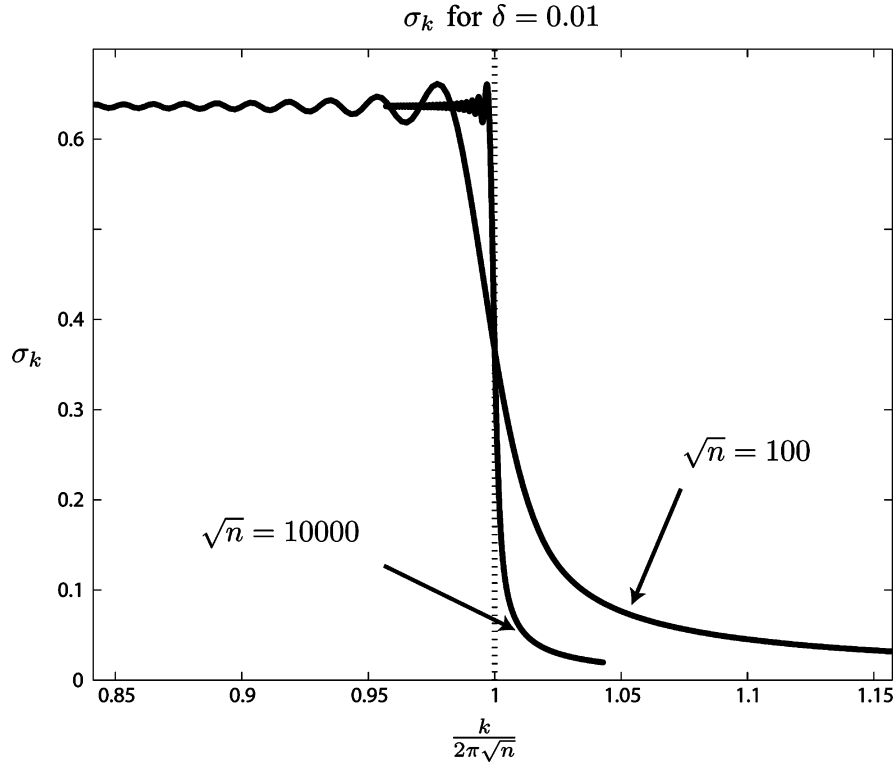


Fig. 4. Plot showing the phase transition of the singular values σ_k .

There exists an $N_0 = O(\sqrt{n} \log n)$, such that for all $\mathbf{r}_m \in \mathcal{M}$ we have

$$\lim_{n \rightarrow \infty} \|E(\mathbf{r}_m) - \hat{E}_{N_0}(\mathbf{r}_m)\|^2 = 0. \quad (22)$$

Some remarks are now in order. The theorem shows that the electric field on \mathcal{M} can be represented using $O(\sqrt{n} \log n)$ functions as n tends to infinity. The $\log n$ factor ensures that we are sufficiently far from the critical value k_c , so that the singular values corresponding to the tail of the field decomposition are essentially zero and the field can be reconstructed with vanishing error.

B. Step Two: The Presence of Scatterers

In this section we show that the field outside \mathcal{M} can be represented using $O(\sqrt{n} \log n)$ basis functions, even when scattering elements are present in the domain. This result has a simple physical interpretation in terms of an *information conservation principle*, which relies on the electromagnetic uniqueness theorem. The uniqueness theorem ensures that the electric field at any point outside \mathcal{M} is uniquely determined by the field on \mathcal{M} . This is composed by the field radiated by D , which by Theorem 3.1 we know has a limited number of degrees of freedom, and by the field backscattered from outside \mathcal{M} , which does not provide any additional information since \mathcal{M} is a closed curve capturing the whole information flow coming out of D . Next, we place this simple intuition into a more rigorous framework.

The electric field at any point $\mathbf{r}_y \in A \setminus V$, and in particular at the receiving antennas, is given by the superposition of two field vectors, denoted E_D and E_S , representing the field due to the currents inside and outside \mathcal{M} , respectively. Formally

$$E(\mathbf{r}_y) = E_D(\mathbf{r}_y) + E_S(\mathbf{r}_y), \quad \mathbf{r}_y \in A \setminus V. \quad (23)$$

We show that both field vectors in (23) can be represented using $O(\sqrt{n} \log n)$ basis functions, as $n \rightarrow \infty$.

Let us first focus on E_D , i.e., the field vector due to the source currents and to the induced currents inside \mathcal{M} . The induced currents are due to the scattered field inside D , and also to the field backscattered from outside D . Since in the analysis of Section III-A the current density in D was assumed arbitrary, the same analysis applies here, by including in $I(\mathbf{r}_d)$ the currents induced by the backscattered field. Thus, by the same steps leading to (15) we can now write the field E_D at a point \mathbf{r}_y outside \mathcal{M} as

$$E_D(\mathbf{r}_y) = \sum_{k=-\infty}^{\infty} \sigma_k \frac{H_k^{(2)}(\beta|\mathbf{r}_y|)}{H_k^{(2)}(2\pi(\sqrt{n} + \delta))} \langle I, v_k \rangle_{L^2} u_k(\mathbf{r}_y). \quad (24)$$

By (15) it follows that (24) also corresponds to the field due to the currents inside D at the point $\mathbf{r}_m \in \mathcal{M}$ with $\angle \mathbf{r}_m = \angle \mathbf{r}_y$, having scaled each harmonic by the factor $H_k^{(2)}(\beta|\mathbf{r}_y|)/H_k^{(2)}(2\pi(\sqrt{n} + \delta))$. Then, using (13) we conclude that there exists a linear operator \mathcal{D}_f such that

$$E_D(\mathbf{r}_y) = (\mathcal{D}_f \circ \mathcal{G}I)(\mathbf{r}_y), \quad \mathbf{r}_y \in A \setminus V. \quad (25)$$

We now focus on E_S , the field vector at the receiving antennas due to the currents induced on the scatterers outside \mathcal{M} . We show that E_S is linearly related to the field on the scatterers, and that this field is in turn linearly related to the currents inside D .

Let $\mathcal{S} \subseteq (A \setminus V)$ denote the domain occupied by the scattering elements outside \mathcal{M} , and let $I(\mathbf{r}_s)$ denote the induced current

density on \mathcal{S} . The functional relationship between $I(\mathbf{r}_s)$ and E_S is given by (11), where we integrate over \mathcal{S} , in lieu of D . Thus

$$E_S(\mathbf{r}_y) = \frac{-\beta^2}{4\omega\epsilon_0} \int_{\mathcal{S}} I(\mathbf{r}_s) H_0^{(2)}(\beta|\mathbf{r}_y - \mathbf{r}_s|) ds, \quad \mathbf{r}_y \in A \setminus V. \quad (26)$$

By Maxwell equations, we can write $I(\mathbf{r}_s)$ in terms of the electric field on \mathcal{S} as follows:

$$I(\mathbf{r}_s) = j\omega(\epsilon(\mathbf{r}_s) - \epsilon_0)E(\mathbf{r}_s), \quad \mathbf{r}_s \in \mathcal{S} \quad (27)$$

wherein $\epsilon(\mathbf{r}_s)$ is the permittivity of the dielectric material² at \mathbf{r}_s . Substituting (27) into (26) we obtain

$$E_S(\mathbf{r}_y) = \frac{-j\beta^2}{4\epsilon_0} \int_{\mathcal{S}} (\epsilon(\mathbf{r}_s) - \epsilon_0)E(\mathbf{r}_s) \times H_0^{(2)}(\beta|\mathbf{r}_y - \mathbf{r}_s|) ds, \quad \mathbf{r}_y \in A \setminus V \quad (28)$$

which shows that E_S is linearly related to the field on \mathcal{S} .

Substituting (28) into (23) we obtain that the field on \mathcal{S} is given by the solution of the following integral equation:

$$E(\mathbf{r}_s) = E_D(\mathbf{r}_s) + \frac{-j\beta^2}{4\epsilon_0} \int_{\mathcal{S}} (\epsilon(\mathbf{r}_s) - \epsilon_0)E(\mathbf{r}_s) \times H_0^{(2)}(\beta|\mathbf{r}_s - \mathbf{r}'|) ds', \quad \mathbf{r}_s \in \mathcal{S}. \quad (29)$$

This is an inhomogeneous Fredholm integral equation of the second kind, whose solution leads to the Liouville-Neumann series. More important for us is that (29) shows a linear relationship between E_D and the field on \mathcal{S} . Since we have already shown in (28) that the field on \mathcal{S} is linearly related to E_S , it now follows that E_S and E_D are also linearly related. Finally, by using (25) we conclude that there exists a linear operator \mathcal{D}_s , such that

$$E_S(\mathbf{r}_y) = (\mathcal{D}_s \circ \mathcal{G}I)(\mathbf{r}_y), \quad \mathbf{r}_y \in A \setminus V. \quad (30)$$

Putting things together, we conclude from (23), (25) and (30) that the electric field at the receiving antennas placed in $A \setminus V$ can be expressed as the superposition of two field vectors. Each of these lies in a Hilbert space whose dimension is limited by the rank of the radiation operator \mathcal{G} and hence can be represented with $O(\sqrt{n} \log n)$ basis functions, as n tends to infinity.

C. Step Three: Back to Information Theory

The input–output relationship between the electromagnetic field at the output of each receiving antenna and the current densities in D can be expressed, in functional form, as

$$E(\mathbf{r}_y) = (\mathcal{D} \circ \mathcal{G}I)(\mathbf{r}_y), \quad \mathbf{r}_y \in A \setminus V \quad (31)$$

where $\mathcal{D} = \mathcal{D}_f + \mathcal{D}_s$. The values in (31) can be stack in a vector \mathbf{O} , whose d th component indicates the intensity of the electric field at receiving node $d \in \mathcal{P} \cap (A \setminus V)$.

The voltage at each receiving antenna is proportional to the intensity of the field at the antenna and is corrupted by some

additive electric noise, which is assumed Gaussian and uncorrelated across antennas. Thus, the voltage values detected by the receiving nodes can be written as

$$\mathbf{Y}_{A \setminus V} = K\mathbf{O} + \mathbf{Z}_{A \setminus V} \quad (32)$$

where K is a constant, and $\mathbf{Z}_{A \setminus V}$ is the Gaussian noise vector. Finally, the input–output relationship between symbols sent by nodes in $\mathcal{P} \cap D$ and received by nodes in $\mathcal{P} \cap (A \setminus V)$ at time i is given by

$$\mathbf{Y}_{A \setminus V}[i] = \mathbf{H}^{(A \setminus V)} \mathbf{X}_D[i] + \mathbf{Z}_{A \setminus V}[i]. \quad (33)$$

where $\mathbf{H}^{(A \setminus V)}$ is given by the composition of the linear operators \mathcal{D} , \mathcal{G} , \mathcal{F} , and the scalar K . It follows from the analysis in the previous section that the rank of $\mathbf{H}^{(A \setminus V)}$ is limited by the rank of \mathcal{G} . Thus, we obtain

$$\text{rank}(\mathbf{H}^{(A \setminus V)}) = O(\sqrt{n} \log n) \quad (34)$$

which proves (8).

IV. EXTENSION TO THREE-DIMENSIONAL NETWORKS

In this section we consider networks in which nodes are located according to a Poisson point process of unit density inside a sphere \mathcal{B}_n of radius $(2n)^{1/3}$. As before, points of the Poisson process are paired uniformly at random. Assuming that each node generates at most P watts, we show that w.h.p. all nodes can (simultaneously) communicate to their intended destinations at rate

$$R(n) = O\left(\frac{(\log n)^3}{n^{1/3}}\right). \quad (35)$$

The proof follows the same steps as in the two-dimensional case, with some minor differences that we outline below. We partition \mathcal{B}_n into two equal parts by drawing a spherical cut of radius $n^{1/3}$ at the origin, which divides \mathcal{B}_n into the inner sphere D and the outer spherical annulus A . Since an order of n communication requests have to cross the boundary of the partition, as before we first study the sum C_n of the rates that can be sent from the transmitters in D to the receivers in A , and then divide C_n by n to obtain the per-node rate R_n . We divide A into an inner and an outer part, denoted by V and $A \setminus V$ respectively, by drawing a sphere of radius $n^{1/3} + \delta$. The total information flow from D to A is decomposed into two contributions:

$$C_n \leq C_V + C_{A \setminus V} \quad (36)$$

wherein C_V and $C_{A \setminus V}$ represent the information flow from D to V and from D to $A \setminus V$, respectively.

By summing the capacities of the individual MISO channels between the nodes in D and each receiver in V , we have, w.h.p.

$$C_V = O(n^{2/3} \log n). \quad (37)$$

On the other hand, $C_{A \setminus V}$ is limited by the number of spatial degrees of freedom, which are $O(n^{2/3}(\log n)^2)$. As a consequence, we have that, w.h.p.

$$C_{A \setminus V} = O(n^{2/3}(\log n)^3). \quad (38)$$

Combining (36), (37) and (38), and dividing by n , (35) follows.

²The analysis in the case of metallic scatterers is completely equivalent.

As before, a proof of (38) is obtained by studying the physics of the information flow from D to $A \setminus V$. There are some geometrical differences that we outline below. Assume that the sources are arbitrarily located in D and can generate an arbitrary current density $I(\mathbf{r}_d)$ [A/m^3], $\mathbf{r}_d \in D$, polarized in the \hat{z} direction.

The electric field radiated by the currents in D and observed on the surface \mathcal{M} separating A from $A \setminus V$ is given by

$$\mathbf{E}(\mathbf{r}_m) = -j\omega\mu\mathbf{A}(\mathbf{r}_m) + \frac{1}{j\omega\epsilon_0}\nabla(\nabla \cdot \mathbf{A}(\mathbf{r}_m)) \quad (39)$$

$$A_z(\mathbf{r}_m) = \frac{1}{4\pi} \int_D \frac{e^{-j\beta|\mathbf{r}_m - \mathbf{r}_d|}}{|\mathbf{r}_m - \mathbf{r}_d|} I(\mathbf{r}_d) d\mathbf{r}_d, \quad \mathbf{r}_m \in \mathcal{M} \quad (40)$$

wherein \mathbf{A} denotes the magnetic vector potential [15], and A_z denotes its z component. The integral kernel in (40) can be decomposed into the sum of spherical harmonics [16, p. 742], yielding

$$A_z(\mathbf{r}_m) = -j\beta \sum_{k=0}^{\infty} \sum_{i=-k}^k h_k^{(2)}(2\pi(n^{1/3} + \delta)) \times Y_{k,i}(\theta_m, \phi_m) \langle I, j_k Y_{k,i} \rangle_{L_2} \quad (41)$$

wherein $\mathbf{r}_m \in \mathcal{M}$ has spherical coordinates $((n^{1/3} + \delta)\lambda, \theta_m, \phi_m)$, j_k is the spherical Bessel function of the first kind and order k , $h_k^{(2)}$ is the spherical Hankel function of second kind and order k , and $Y_{k,i}$ is the (k th, i th) spherical harmonic function. The Hilbert–Schmidt decomposition of (41) can be written as

$$A_z(\mathbf{r}_m) = \sum_{k=0}^{\infty} \sigma_k \sum_{i=-k}^k \langle I, v_{k,i} \rangle_{L^2} u_{k,i}(\mathbf{r}_m), \quad \mathbf{r}_m \in \mathcal{M} \quad (42)$$

wherein, for $\mathbf{r}_m = ((n^{1/3} + \delta)\lambda, \theta_m, \phi_m) \in \mathcal{M}$

$$u_{k,i}(\mathbf{r}_m) = \frac{h_k^{(2)}(2\pi(n^{1/3} + \delta)) Y_{k,i}(\theta_m, \phi_m)}{(n^{1/3} + \delta) |h_k^{(2)}(2\pi(n^{1/3} + \delta))|} \quad (43)$$

while, for $\mathbf{r}_d = (r_d, \theta_d, \phi_d) \in D$,

$$v_{k,i}(\mathbf{r}_d) = -j \frac{j_k(\beta r_d) Y_{k,i}(\theta_d, \phi_d)}{\left(\int_0^{\lambda n^{1/3}} |j_k(\beta r_d)|^2 r_d^2 dr_d \right)^{1/2}} \quad (44)$$

and

$$\sigma_k = \beta \left| h_k^{(2)}(2\pi(n^{1/3} + \delta)) \right| (n^{1/3} + \delta) \times \left(\int_0^{\lambda n^{1/3}} |j_k(\beta r_d)|^2 r_d^2 dr_d \right)^{1/2}. \quad (45)$$

Evaluating the integral in (45) using identity (5.54.2) of [13], and writing the spherical Bessel functions in terms of cylindrical Bessel functions of fractional order using identities in [13, par. 10.1.1], we obtain

$$\begin{aligned} \sigma_k &= \frac{\pi\sqrt{\lambda}}{\sqrt{8}} n^{1/3} \sqrt{n^{1/3} + \delta} \\ &\times \left| H_{k+1/2}^{(2)}(2\pi(n^{1/3} + \delta)) \right| \times ((J_{k+1/2}(2\pi n^{1/3}))^2 \\ &- J_{k-1/2}(2\pi n^{1/3}) J_{k+3/2}(2\pi n^{1/3}))^{1/2}. \end{aligned} \quad (46)$$

Let us compare (46) and (21). The two equations have the same asymptotic behavior, provided that in (21) we replace $n^{1/2}$ with

$n^{1/3}$, *ceteris paribus*. By following exactly the same steps as in the proof of Theorem 1 and using (42), it then follows that there exists an $N_0 = O(n^{1/3} \log n)$, such that A_z can be represented with vanishing error as $n \rightarrow \infty$ using

$$\sum_{k=0}^{N_0} (1 + 2k) = O(n^{2/3} (\log n)^2)$$

singular functions. We have assumed so far that the current density inside D was arbitrary, but polarized in the \hat{z} direction. By symmetry, the analysis in the cases of polarization in the \hat{x} and \hat{y} directions is equivalent and, by the superposition of the effects, the general case of arbitrary polarization can lead up to a three-fold increase in the degrees of freedom. However, since an arbitrary electromagnetic field in an homogeneous source-free space can be obtained by superposition of Transverse Electric and Transverse Magnetic solutions, and since both of them can be represented in terms of spherical harmonics [15, pp. 129–131, p. 267], the increase is only two-fold in case of arbitrary polarization. In any case, the order result $O(n^{2/3} (\log n)^2)$ does not change in the case of arbitrary polarization. On the other hand, (39) shows that the electric field \mathbf{E} and A_z are related through a linear operator, so \mathbf{E} can also be represented with a vanishing error using $O(n^{2/3} (\log n)^2)$ basis functions, as $n \rightarrow \infty$. Next, proceeding exactly as in Section III-B, it follows that the presence of scattering objects in $A \setminus V$ does not increase the number of degrees of freedom of the field at the receiving antennas. Finally, (38) is obtained as before, by applying the information-theoretic cut-set bound and assuming to beamform the total transmitted power in each of the $O(n^{2/3} (\log n)^2)$ spatial channels between transmitters and receivers.

V. LINEAR CAPACITY SCALING

The objective of the network engineer is to design wireless systems which fully exploit the number of degrees of freedom available for communication. With a successful design, and if the number of degrees of freedom scales linearly with the number of nodes, then more and more users can be added to the network without sacrificing performance and the engineer fulfills the dream of achieving linear capacity scaling. A recent paper of Özgür, Lévêque and Tse [28] almost fulfilled this dream. The authors assume a stochastic fading channel model in which all emitted signals are received with independent phases, which leads to a number of spatial degrees of freedom that scales linearly with the number of nodes. Then, they propose an ingenious node cooperation protocol which exploits these degrees of freedom, and allows to maintain an almost constant per-node bit rate as the network's size scales, when the path loss is sufficiently low.

However, we have shown that the number of spatial degrees of freedom cannot be assumed to grow linearly with the number of nodes, but in 2-D it is limited by the spatial length of the cut that divides the network into two halves, so it can grow at most as \sqrt{n} ; and in 3-D it is limited by the surface of the cut, growing as $n^{2/3}$. Hence, space can be viewed as a *capacity bearing object* which poses a fundamental limit on the achievable information rate, independent of path loss assumptions. An intuitive picture of this is as follows. Each communication channel can

be viewed as occupying a unit of space along the cut through which the information must flow. Sharing this limited spatial resource among all the nodes leads to our capacity bounds.

Given this limitation, we are led to the following engineering guideline: geometry should play a key role in the design of the network, hand in hand with protocol development. While previously proposed cooperation strategies are not tied to the geometric configuration and dimensionality of the network, with a careful geometric design the spatial resource can be carefully allocated to the users of the network, and then exploited by the communication protocol. For example, one could try to design *sparse networks* in which the number of nodes is small compared to the spatial resource available for communication and investigate whether this spatial resource can be exploited in practice through node cooperation. One such configuration could be a network in which the nodes are confined to a two-dimensional space, while propagation and scattering occurs in all three dimensions. We wish to investigate these issues in a forthcoming paper, whose seeds are in [11], and shall not discuss them further here.

Looking in retrospective, we also see that the results reported in the present paper are of similar flavor as the ones obtained for point-to-point multiple antenna arrays in [18], [21], [33], and [35], where physical arguments have been used to challenge the original optimistic results reported in the celebrated works of Foschini [8] and Telatar [36]. This challenge has also been supported by experimental evidence that the mutual coupling between antennas, arising when the spacing between them becomes smaller than the wavelength, does not allow independent signals to be detected at the receivers [7].

To bypass such arguments, it is customary to note that while the above can be of concern in antenna arrays where radiating elements are packed close to each other, in the context of nodes spatially distributed at random on the plane this issue is irrelevant, as nodes are typically in the far field of each other. For example, in a network operating at 3 GHz, the carrier wavelength is 0.1 m, while a reasonable separation distance between nearest neighbor nodes would be of the order of tens of meters, very much beyond the danger of incurring into near field coupling effects! Nevertheless, our results show that this heuristic argument fails in the scaling limit. By the uniqueness theorem, the field on the closed cut considered in our analysis completely determines the signal measured at all the receivers outside the cut and this field has in 2-D only an order of \sqrt{n} degrees of freedom. Therefore, it is not possible to generate an order of n independent signals at the receivers, *even if* all the nodes are in the far field of each other. In other words, the degrees of freedom bottleneck is due to the flow through the cut, rather than to the spacing between the antennas.

As a final remark, we underline that the asymptotic results presented in this paper cannot be directly applied to fixed size networks, for which capacity can be limited by numerous other factors. Position and properties of the different objects in the environment that are responsible for reflection, diffraction, scattering, and absorption of the propagating waves play an important role in determining the number of available spatial degrees of freedom, while the results presented in this paper hold *uniformly* over all possible propagation environments, having

fixed the dimensionality of the space, and in the limit of large networks. For this reason, the question of when the geometric limitations showed here become of practical relevance does not appear to have a unique answer. For small 2-D networks, the number of available degrees of freedom in a rich scattering environment can be much larger than n , before eventually reaching its asymptotic $O(\sqrt{n})$ value as the network grows larger. In contrast, in an environment dominated by absorption the number of available degrees of freedom can be as small as zero, when communication is shaded by large absorbing obstacles.

To conclude, we are still far from reaching the holy grail of information theory for wireless communication, and the mathematical characterization of the capacity region of any fixed-size network remains “*a hope beyond the shadow of a dream.*”

APPENDIX I PROOF OF THEOREM 3.1

From (15) we have that, for any $\mathbf{r}_m \in \mathcal{M}$

$$\begin{aligned} & \left\| E(\mathbf{r}_m) - \hat{E}_{N_0}(\mathbf{r}_m) \right\|^2 \\ & \leq \left\| \sum_{k=-\infty}^{-N_0} \sigma_k \langle I, v_k \rangle_{L^2} u_k(\mathbf{r}_m) \right\|^2 \\ & \quad + \left\| \sum_{k=N_0}^{\infty} \sigma_k \langle I, v_k \rangle_{L^2} u_k(\mathbf{r}_m) \right\|^2 \\ & \leq \sum_{k=-\infty}^{-N_0} |\sigma_k|^2 \langle I, I \rangle_{L^2} + \sum_{k=N_0}^{\infty} |\sigma_k|^2 \langle I, I \rangle_{L^2} \\ & \leq \frac{2nP}{a} \sum_{k=N_0}^{\infty} |\sigma_k(\sqrt{n} + \delta)|^2 \end{aligned} \quad (47)$$

where the first inequality follows from the triangle inequality; the second inequality follows from the fact that u_k and v_k have unit norm and from the Cauchy–Schwarz inequality; the third inequality follows from $\sigma_k = \sigma_{-k}$ (due to the symmetry of Bessel functions of integer order) and the power constrain in (12). Thus, in order to prove the theorem, it suffices to show that there exists an $N_0 = O(\sqrt{n} \log n)$, such that

$$\lim_{n \rightarrow \infty} n \sum_{k=N_0}^{\infty} |\sigma_k(\sqrt{n} + \delta)|^2 = 0. \quad (48)$$

Using the recurrence formulas [1, identity 9.1.27] we can relate the Bessel functions of order $k-1$ and $k+1$ to the corresponding Bessel functions of order k , as follows:

$$\begin{aligned} J_{k-1}(2\pi\sqrt{n}) &= \frac{k}{2\pi\sqrt{n}} J_k(2\pi\sqrt{n}) + J'_k(2\pi\sqrt{n}) \\ J_{k+1}(2\pi\sqrt{n}) &= \frac{k}{2\pi\sqrt{n}} J_k(2\pi\sqrt{n}) - J'_k(2\pi\sqrt{n}) \end{aligned}$$

wherein $J'_k(x)$ denotes the derivative of the Bessel function with respect to the argument x . Thus

$$\begin{aligned} & J_{k-1}(2\pi\sqrt{n}) J_{k+1}(2\pi\sqrt{n}) \\ &= \frac{k^2}{4\pi^2 n} (J_k(2\pi\sqrt{n}))^2 - (J'_k(2\pi\sqrt{n}))^2. \end{aligned} \quad (49)$$

Substituting (49) into (21), the singular values can be written as

$$\sigma_k(\sqrt{n} + \delta) = \frac{\sqrt{\mu_0 2n}}{2\sqrt{\epsilon_0}} \pi^2 (\sqrt{n} + \delta)^{1/2} \left| H_k^{(2)}(2\pi(\sqrt{n} + \delta)) \right| \times \left((J'_k(2\pi\sqrt{n}))^2 - (k^2/(4\pi^2 n) - 1)(J_k(2\pi\sqrt{n}))^2 \right)^{1/2} \quad (50)$$

where we emphasize the dependence of the singular values on the radius of the circle \mathcal{M} . Observe that $(k^2/(4\pi^2 n) - 1) \geq 0$ for all $k \geq 2\pi\sqrt{n}$. Thus, from (50) it follows that, for $k \geq 2\pi\sqrt{n}$

$$|\sigma_k(\sqrt{n} + \delta)|^2 = O \left(n^{3/2} \left| H_k^{(2)}(2\pi(\sqrt{n} + \delta)) \right|^2 \times \left| J'_k(2\pi\sqrt{n}) \right|^2 \right) \quad (51)$$

as $n \rightarrow \infty$.

Next, we use Olver's uniform asymptotic expansions for Bessel functions [29], [30] to bound the right-hand side of (51). Notice that, while the Hankel function $|H_k^{(2)}(2\pi(\sqrt{n} + \delta))|$ is exponentially increasing in k , the derivative of the Bessel function $|J'_k(2\pi\sqrt{n})|$ is exponentially decreasing in k . In the following, by studying the rate of growth and decay of the two functions, we conclude that the singular values decrease exponentially to zero as k approaches infinity.

Let z denote the ratio between the argument and the order of $J'_k(2\pi\sqrt{n})$, i.e., $z = \frac{2\pi\sqrt{n}}{k}$. Identity [30, eq. (5.10)] and the triangle inequality yield, for $0 < z \leq 1$

$$|J'_k(kz)| \leq \frac{2}{k^{2/3}z} \left(\frac{1-z^2}{4\zeta(z)} \right)^{1/4} \left[\frac{\text{Ai}(k^{2/3}\zeta(z))}{k^{2/3}} + |\text{Ai}'(k^{2/3}\zeta(z))| + |\eta(k, z)| + \frac{|\epsilon(k, z)|}{k^{2/3}} \right] \quad (52)$$

wherein Ai denotes the Airy function, for $0 < z \leq 1$ the function $\zeta(z)$ is defined as

$$\frac{2}{3}\zeta^{3/2}(z) = \int_z^1 \frac{\sqrt{1-u^2}}{u} du = \log \left(\frac{1 + \sqrt{1-z^2}}{z} \right) - \sqrt{1-z^2} \quad (53)$$

and $|\epsilon(k, z)|$ and $|\eta(k, z)|$ are subject to the following bounds [30, Sec. 5]:

$$|\epsilon(k, z)| \leq k^{-1} \text{Ai}(k^{2/3}\zeta(z)) \quad (54)$$

$$|\eta(k, z)| \leq k^{-1} \text{Ai}(k^{2/3}\zeta(z)). \quad (55)$$

Substituting (54) and (55) into (52), and using $\text{Ai}(x)/|\text{Ai}'(x)| \leq 2$, which holds for all $x \geq 0$ [30, page 11], we obtain that, for $0 < z \leq 1$

$$|J'_k(kz)| \leq \frac{14}{k^{2/3}z} \left(\frac{1-z^2}{4\zeta(z)} \right)^{1/4} |\text{Ai}'(k^{2/3}\zeta(z))|. \quad (56)$$

Equation (56) provides a bound (uniform in $0 < z \leq 1$) for $|J'_k(kz)|$ in terms of the derivative of the Airy function. We now want to find a similar bound for the Hankel function. We start by noticing that

$$\left| H_k^{(2)}(x) \right| \leq |J_k(x)| + |Y_k(x)| \quad (57)$$

wherein $Y_k(x)$ is the Bessel function of the second kind and order k . Let z_δ denote the ratio between the argument and the order of $H_k^{(2)}(2\pi(\sqrt{n} + \delta))$, i.e., $z_\delta = (2\pi\frac{\sqrt{n} + \delta}{k}) = z + \frac{2\pi\delta}{k}$. By identities [1, eq. (9.3.6)], we have that, for $0 < z_\delta \leq 1$

$$J_k(kz_\delta) = \left(\frac{4\zeta(z_\delta)}{1-z_\delta^2} \right)^{1/4} \left[\frac{\text{Ai}(k^{2/3}\zeta(z_\delta))}{k^{1/3}} + \frac{e^{-2/3k\zeta^{3/2}(z_\delta)}}{1+k^{1/6}\zeta^{1/4}(z_\delta)} O \left(\frac{1}{k^{4/3}} \right) \right] \quad (58)$$

and

$$Y_k(kz_\delta) = - \left(\frac{4\zeta(z_\delta)}{1-z_\delta^2} \right)^{1/4} \left[\frac{\text{Bi}(k^{2/3}\zeta(z_\delta))}{k^{1/3}} + \frac{e^{+2/3k\zeta^{3/2}(z_\delta)}}{1+k^{1/6}\zeta^{1/4}(z_\delta)} O \left(\frac{1}{k^{4/3}} \right) \right]. \quad (59)$$

Thus, putting together (57), (58), and (59), we also have a bound (uniform in $0 < z_\delta \leq 1$) for the Hankel function in terms of the Airy functions Ai and Bi. The next step is to provide exponential bounds for the Airy functions.

We have, for $k^{2/3}\zeta \geq 1$ [31, p. 394]

$$\begin{aligned} \text{Ai}(k^{2/3}\zeta) &\leq \frac{e^{-\frac{2}{3}k\zeta^{3/2}}}{k^{1/6}\zeta^{1/4}} \\ |\text{Ai}'(k^{2/3}\zeta)| &\leq k^{1/6}\zeta^{1/4} e^{-\frac{2}{3}k\zeta^{3/2}} \\ \text{Bi}(k^{2/3}\zeta) &\leq \frac{e^{+\frac{2}{3}k\zeta^{3/2}}}{k^{1/6}\zeta^{1/4}}. \end{aligned} \quad (60)$$

By (53), we notice that $\zeta(z)$ is a decreasing function of z , which tends to infinity as $z \rightarrow 0^+$ and is 0 when $z = 1$. Hence, $k^{2/3}\zeta(\frac{2\pi\sqrt{n}}{k}) \geq 1$, which is required for (60) to hold, is not satisfied when k is close to the critical value $2\pi\sqrt{n}$. However, by choosing $k \geq 2\pi\sqrt{n} \log n$ the desired condition holds for n large.

Thus, substituting (60) into (56), (58), and (59), it follows that, for $k \geq 2\pi\sqrt{n} \log n$

$$\begin{aligned} &\left| H_k^{(2)}(2\pi(\sqrt{n} + \delta)) \right| \\ &= O \left(\frac{1}{k^{1/2}(1-z_\delta^2)^{1/4}} e^{+\frac{2}{3}k\zeta^{3/2}(z_\delta)} \right) \end{aligned} \quad (61)$$

$$\begin{aligned} &\left| J'_k(2\pi(\sqrt{n})) \right| \\ &= O \left(\frac{(1-z^2)^{1/4}}{k^{1/2}z} e^{-\frac{2}{3}k\zeta^{3/2}(z)} \right) \end{aligned} \quad (62)$$

as $n \rightarrow \infty$.

Notice that $\zeta(z_\delta) < \zeta(z)$, since $|z_\delta| > |z|$ for any $\delta > 0$. As a consequence, the rate of growth of the exponential in (61) is smaller than the rate of decay of the exponential in (62). Substituting (61) and (62) into (51), and using the fact that $(1-z^2)/(1-z_\delta^2) = O(1)$ as $n \rightarrow \infty$, we obtain that for $k \geq 2\pi\sqrt{n} \log n$

$$\begin{aligned} &|\sigma_k(\sqrt{n} + \delta)|^2 \\ &= O \left(\sqrt{n} \exp \left\{ -\frac{4}{3}k \left[\zeta^{3/2} \left(\frac{2\pi\sqrt{n}}{k} \right) \right] \right\} \right) \end{aligned}$$

$$-\zeta^{3/2} \left(\frac{2\pi\sqrt{n}}{k} + \frac{2\pi\delta}{k} \right) \Big] \Big] \Big] \quad (63)$$

as $n \rightarrow \infty$.

Let us focus on the exponent in the right-hand side of (63). By (53), we have

$$\begin{aligned} & -2k \left[\frac{2}{3} \zeta^{3/2} \left(\frac{2\pi\sqrt{n}}{k} \right) - \frac{2}{3} \zeta^{3/2} \left(\frac{2\pi\sqrt{n}}{k} + \frac{2\pi\delta}{k} \right) \right] \\ &= -2k \int_{\frac{2\pi\sqrt{n}}{k}}^{\frac{2\pi\sqrt{n}}{k} + \frac{2\pi\delta}{k}} \frac{\sqrt{1-u^2}}{u} du \\ &\leq -2k \int_{\frac{2\pi\sqrt{n}}{k}}^{\frac{2\pi\sqrt{n}}{k} + \frac{2\pi\delta}{k}} \left(\frac{1}{u} - 1 \right) du \\ &= -2k \log \left(1 + \frac{\delta}{\sqrt{n}} \right) + 4\pi\delta \end{aligned} \quad (64)$$

where the inequality follows from $\sqrt{1-u^2} \geq 1-u$, for all $u \in [0, 1]$. Substituting (64) into (63) it follows that, for all $\delta > 0$ and for all $k \geq 2\pi\sqrt{n} \log n$

$$|\sigma_k(\sqrt{n} + \delta)|^2 = O \left(\sqrt{n} e^{-2k \log(1 + \frac{\delta}{\sqrt{n}})} \right) \quad (65)$$

as $n \rightarrow \infty$.

Finally, to obtain (48) we choose $N_0 = \max\{\frac{2}{\delta}, 2\pi\}\sqrt{n} \log n$ and use the bound (65), which is uniform in $k \geq \max\{\frac{2}{\delta}, 2\pi\}\sqrt{n} \log n$. Hence, for the choice of N_0 above, there exists a uniform constant C , such that as $n \rightarrow \infty$, we have

$$\begin{aligned} & n \sum_{k=N_0}^{\infty} |\sigma_k(\sqrt{n} + \delta)|^2 \\ &= n \sum_{k=\max\{\frac{2}{\delta}, 2\pi\}\sqrt{n} \log n}^{\infty} |\sigma_k(\sqrt{n} + \delta)|^2 \\ &\leq n^{3/2} \sum_{k=\max\{\frac{2}{\delta}, 2\pi\}\sqrt{n} \log n}^{\infty} C e^{-2k \log(1 + \frac{\delta}{\sqrt{n}})} \\ &= n^{3/2} \frac{C \left(e^{-2 \log(1 + \frac{\delta}{\sqrt{n}})} \right)^{\max\{\frac{2}{\delta}, 2\pi\}\sqrt{n} \log n}}{1 - e^{-2 \log(1 + \frac{\delta}{\sqrt{n}})}} \\ &\leq \left(\frac{n^{5/2}}{2\delta\sqrt{n} + \delta^2} + n^{3/2} \right) C e^{-\log n^4 [\frac{\sqrt{n}}{\delta} \log(1 + \frac{\delta}{\sqrt{n}})]} \\ &= o \left(\frac{1}{n^2} \right) \\ &\rightarrow 0 \end{aligned}$$

which concludes the proof. \square

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REFERENCES

- [1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1972.
- [2] S. Aeron and V. Saligrama, "Wireless ad hoc networks: Strategies and scaling laws for the fixed SNR regime," *IEEE Trans. Inf. Theory*, vol. IT-53, no. 6, pp. 2044–2059, Jun. 2007.
- [3] S. Ahmad, A. Jovičić, and P. Viswanath, "Outer bounds to the capacity region of wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-52, no. 6, pp. 2770–2776, Jun. 2006.
- [4] O. M. Bucci and G. Franceschetti, "On the spatial bandwidth of scattered fields," *IEEE Trans. Antennas Propag.*, vol. AP-35, no. 12, pp. 1445–1455, Dec. 1987.
- [5] O. M. Bucci and G. Franceschetti, "On the degrees of freedom of scattered fields," *IEEE Trans. Antennas Propag.*, vol. AP-37, no. 7, pp. 918–926, Jul. 1989.
- [6] T. Cover and J. Thomas, *Elements of Information Theory*. New York: Wiley, 2006.
- [7] D. W. Browne, M. Manteghi, M. P. Fitz, and Y. Rahmat-Samii, "Experiments with compact antenna arrays for MIMO radio communications," *IEEE Trans. Antennas Propag.*, vol. AP-54, no. 11, pp. 3239–3250, Nov. 2006.
- [8] G. J. Foschini, "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.
- [9] M. Franceschetti, "A note on Lévêque and Telatar's upper bound on the capacity of wireless ad-hoc networks," *IEEE Trans. Inf. Theory*, vol. IT-53, no. 9, pp. 3207–3211, Sep. 2007.
- [10] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Inf. Theory*, vol. IT-53, no. 3, pp. 1009–1018, Mar. 2007.
- [11] M. Franceschetti, M. D. Migliore, and P. Minero, "The degrees of freedom of wireless networks: Information theoretic and physical limits," in *Proc. Allerton Conf. Commun. Computing Contr.*, Monticello, IL, Sep. 2008.
- [12] R. Gowaikar, B. Hochwald, and B. Hassibi, "Communication over a wireless network with random connections," *IEEE Trans. Inf. Theory*, vol. IT-52, no. 7, pp. 2857–2871, Jul. 2006.
- [13] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, A. Jeffrey, Ed. New York: Academic, 1994.
- [14] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-42, no. 2, pp. 388–404, Mar. 2000.
- [15] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.
- [16] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. New York: Wiley, 1962.
- [17] A. Jovičić, P. Viswanath, and S. R. Kulkarni, "Upper bounds to transport capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-50, no. 11, pp. 2555–2565, Nov. 2004.
- [18] R. A. Kennedy, P. Sadeghi, T. D. Abhayapala, and H. M. Jones, "Intrinsic limits of dimensionality and richness in random multipath fields," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2542–2556, Jun. 2007.
- [19] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling in wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-50, no. 11, pp. 1041–1049, Jun. 2004.
- [20] O. Lévêque and E. Telatar, "Information theoretic upper bounds on the capacity of large, extended ad-hoc wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-51, no. 3, pp. 858–865, Mar. 2005.
- [21] K. Liu, V. Raghavan, and A. M. Sayeed, "Capacity scaling and spectral efficiency in wideband correlated MIMO channels," *IEEE Trans. Inf. Theory*, vol. IT-49, no. 10, pp. 2504–2526, Oct. 2003.
- [22] M. D. Migliore, "On the role of the number of degrees of freedom of the field in MIMO channels," *IEEE Trans. Antennas Propag.*, vol. AP-54, no. 2, pp. 620–628, Feb. 2006.
- [23] D. A. B. Miller, "Communicating with waves between volumes: Evaluating orthogonal spatial channels and limits on coupling strengths," *Appl. Opt.*, vol. 39, no. 11, pp. 1681–1699, Apr. 2000.
- [24] U. Niesen, P. Gupta, and D. Shah, "On capacity scaling in arbitrary wireless networks," in *Proc. Inf. Theory and Appl. Workshop (ITA)*, San Diego, Univ. Calif., Feb. 2007, pp. 5–.

- [25] U. Niesen, P. Gupta, and D. Shah, "Capacity region of large wireless networks," in *Proc. Allerton Conf. Commun. Computing and Contr.*, Monticello, IL, Sep. 2008.
- [26] A. Özgür, R. Johari, D. N. C. Tse, and O. Lévêque, "Information theoretic operating regimes of large wireless networks," in *Proc. Int. Symp. Inf. Theory (IEEE-ISIT)*, Toronto, Canada, 2008.
- [27] A. Özgür, O. Lévêque, and E. Preissmann, "Scaling laws for one and two-dimensional random wireless networks in the low attenuation regime," *IEEE Trans. Inf. Theory*, vol. IT-53, no. 10, pp. 3573–3585, Oct. 2007.
- [28] A. Özgür, O. Lévêque, and D. N. C. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Trans. Inf. Theory*, vol. IT-53, no. 10, pp. 3549–3572, Oct. 2007.
- [29] F. W. J. Olver, "The asymptotic expansion of Bessel functions of large order," *Philos. Trans. Roy. Soc. London Ser. A*, vol. 247, pp. 328–368, 1954.
- [30] F. W. J. Olver, "Tables for Bessel functions of moderate or large orders," *National Physical Laboratory Mathematical Tables*, vol. 6, 1962, Dep. Scient. Indust. Res., (Her Majesty's Stationery Office), London.
- [31] F. W. J. Olver, "Asymptotics and special functions," in *Reprint*. MA: Wellesley, 1997, AKP Classics. A. K. Peters.
- [32] R. Piestum and D. A. B. Miller, "Electromagnetic degrees of freedom of an optical system," *J. Opt. Soc. Amer.*, vol. 17, no. 5, pp. 892–902, May 2000.
- [33] A. S. Y. Poon, R. W. Brodersen, and D. N. C. Tse, "Degrees of freedom in multiple antenna channels: A signal space approach," *IEEE Trans. Inf. Theory*, vol. IT-51, no. 2, pp. 523–536, Feb. 2005.
- [34] O. M. Bucci, C. Gennarelli, and C. Savarese, "Representation of electromagnetic fields over arbitrary surfaces by a finite and nonredundant number of samples," *IEEE Trans. Antennas Propag.*, vol. 46, pp. 351–359, 1998.
- [35] A. M. Sayeed, V. Raghavan, and J. H. Kotecha, "Capacity of space-time wireless channels: A physical perspective," in *Proc. Inf. Theory Workshop (ITW'04)*, Oct. 2004, pp. 434–439.
- [36] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–596, Nov. 1999.
- [37] G. T. di Francia, "Resolving power and information," *J. Opt. Soc. Amer.*, vol. 45, no. 7, pp. 497–501, Jul. 1955.
- [38] G. Toraldo Di Francia, "Directivity, super-gain and information," *IRE Trans. Antennas Propag.*, vol. AP-4, no. 3, pp. 473–478, Jul. 1956.
- [39] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communications: Scaling laws and optimal operation," *IEEE Trans. Inf. Theory*, vol. 50, no. 5, pp. 748–767, May 2004.
- [40] L.-L. Xie and P. R. Kumar, "On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks," *IEEE Trans. Inf. Theory*, vol. IT-52, no. 6, pp. 2313–2328, Jun. 2006.
- [41] F. Xue, L.-L. Xie, and P. R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Trans. Inf. Theory*, vol. IT-51, no. 3, pp. 834–847, Mar. 2005.

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