

# Impact, drag, and the granular critical state

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## ABSTRACT

The response of dense ensembles of non-cohesive granular media (GM) subject to the motion of an intruder is important in many industrial and biological [1] settings. Previous studies of impact [2] and drag [3, 4, 5] have identified general features of force and flow response, however the role of the initial volume fraction,  $\phi$ , has not been studied in detail. Here we investigate the response of GM perturbed by the motion of single intruders as a function of  $\phi$  in experiment and in soft sphere discrete particle simulation [6, 7, 1]. Comparing the response for experiments with different intruder shape, trajectory, and driving reveals the importance of the critical volume fraction,  $\phi_c$ , the packing density at which grains neither dilate nor consolidate when sheared. Experiments examining impact dynamics of ballistic spheres with GM reveal a pronounced dependence on  $\phi$ . Post-impact crater morphology identifies the critical packing state  $\phi_c$ , and indicates an associated change in spatial response. Existing phenomenological models fail to capture the observed impact force for most  $\phi$ ; only near  $\phi_c$  is force separable into additive terms linear in depth and quadratic in velocity. At fixed depth the quadratic drag coefficient decreases (increases) with depth for  $\phi < \phi_c$  ( $\phi > \phi_c$ ). At fixed low velocity, depth dependence of force shows a Janssen-type exponential response with a length scale that decreases with increasing  $\phi$  and is nearly constant for  $\phi > \phi_c$ . DEM simulations validated by experiment [1] reveal sharp differences in the local flow around the sphere for initial  $\phi$  above and below  $\phi_c$ . As  $\phi$  is increased, the region of upward flow surrounding the penetrating sphere decreases in horizontal extent and the upward grain flux increases. We also study horizontal plate drag at constant velocity to examine the effects of  $\phi$ . A bifurcation in force and flow occurs at the onset of dilatancy  $\phi_c$ . Below  $\phi_c$  there are rapid fluctuations in the drag force  $F_d$ . Above  $\phi_c$  fluctuations in  $F_d$  are periodic and increase in magnitude with  $\phi$ . Velocity field measurements show that the bifurcation in  $F_d$  results from the formation of stable shear bands above  $\phi_c$  which are created and destroyed periodically during drag and which confine the motion of grains to a localized wedge-like region in front of the plate. A friction-based wedge flow model captures the dynamics for  $\phi > \phi_c$ . Together, the impact and the plowing studies highlight the importance of the initial  $\phi$  in determining the response of granular media to an intruder. Whether hydrodynamic models of dense granular flow (e.g. [3]) can account for the variations in flow caused by changes in  $\phi$  remains an open question.

**Acknowledgement.** This work was supported by the Burroughs Wellcome Fund and the ARL MAST CTA under Cooperative Agreement No. W911NF-08-2-0004.

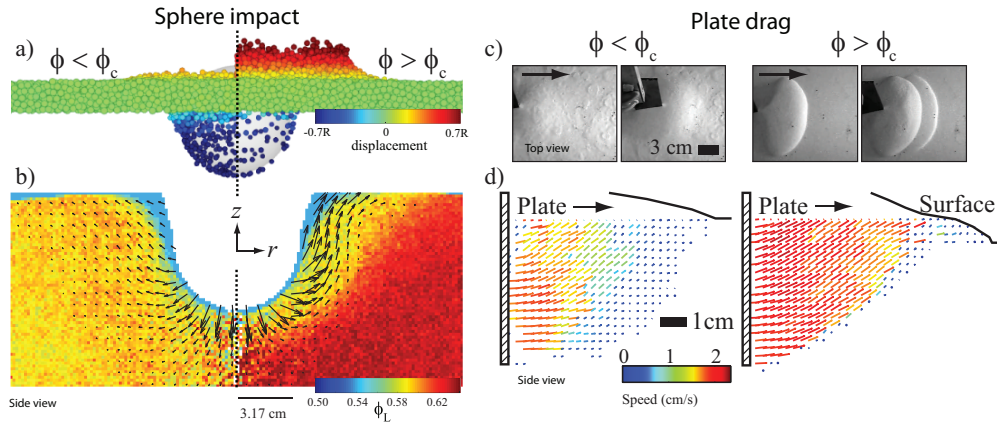


Figure 1: Sensitivity of granular flow past intruders to initial  $\phi$ ; the left (right) side of each sub-figure shows GM response for  $\phi$  below (above)  $\phi_c$ . **Left:** DEM simulations of a sphere impacting 0.294 cm grains with an impact velocity of 1.8 m/s. (a) 3D rendering of particle displacements and (b) azimuthally averaged local volume fraction  $\phi_L$  (color) and velocity field (arrows) show greater flow localization with increasing  $\phi$ . **Right:** Experimental measurements of flow near a horizontally displaced plate reveal (c) smooth surface deformation and (d) a velocity field with small magnitude and extent for  $\phi < \phi_c$ ; for  $\phi > \phi_c$  (c) surface deformations are step-like and (d) the velocity field is larger in magnitude and extent.

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