Geometric cohesion in granular materials

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Stacks made up of rods or staples display many behaviors not seen in the more familiar and better-studied sand piles.

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Collections of granular materials—small particles interacting primarily through contact forces—typically do not cohere unless they are wet, as in sand castles, or are small enough for attractive molecular van der Waals forces to become significant, as in flour. Industrial processes that move large amounts of material—cereal, pharmaceutical tablets, coal, and many others—through silos, hoppers, and conveyor belts rely on the fact that ordinary, round granular particles separate easily.

When the particle shape is radically aspherical, however, the situation is dramatically different. Long, thin rods and U-shaped staples both form rigid, surprisingly tall piles that hold their shape just as a solid would; figure 1 shows a couple of examples. Moreover, the piles resist being pulled apart. They demonstrate a cohesion due solely to the particle geometry—geometric cohesion.

Albert Philipse seems to have been the first to document geometric cohesion. In a 1996 study of objects of various shapes, he observed that when a rod's length-to-width aspect ratio was greater than 35, the rods did not pour freely as sugar does, but rather emerged in a single clump. Philipse also considered a broad size range of particles—from colloids and microscopic silica rods to wood dowels—dropped freely into a container and found that the amount of space occupied by the particles, called the packing fraction, depends on just two quantities: the average number of neighbors with which a particle is in contact and the average amount of space that a particle makes inaccessible to others. That last quantity, the excluded volume, is not the same as the particle volume. For example, the centers of two identical spheres must be separated by at least a diameter D, so the excluded volume is $\frac{4}{3}\pi D^3$.

Transition from granular to solid

Two experiments that I conducted with undergraduate students at the Rochester Institute of Technology carefully explored particulars of how rods stop behaving like ordinary sand and begin acting like a solid as their aspect ratio increases. Ken Desmond and I investigated the drag force exerted by a pile on a small ball pushed upward through the material. That force had been well studied in ordinary granular materials; unlike the constant drag experienced in air or liquid, the granular drag force fluctuates in time as the particles rearrange in a stick-slip manner around the intruder. In our experiment, piles containing large-aspect-ratio rods lifted en masse, as expected. But we also observed a broad range of conditions over which the pile behaved both as a typical granular material and as a solid. The pile would sometimes undergo small rearrangements around the intruder and show a fluctuating force with characteristics similar to those seen in ordinary sand. Occasionally, though, even for aspect ratios as low as about 15, the intruder would lift the material above it, as if that material were a solid, before the pile collapsed around the ball. Given Philipse's observation that packings of rods with aspect ratios less than 30 flow like a liquid, we were surprised to see solid-like behavior at such low aspect ratios.

Undergraduate Melissa Trepanier joined me to explore a different feature of the gradual onset of solidity. Like Desmond, Trepanier extended a canonical granular experiment—in this case, the collapse of cylindrical columns of sand—to large-aspect-ratio rods. In her experiments, we formed a pile of material in a container, suddenly removed the vessel, and measured the length to which the material spread. Geometry predicts a close relation between the runoff distance and the so-called angle of repose that the pile makes when flow stops. For example, if the material can support a 90° angle of repose, then no collapse occurs and the pile maintains its cylindrical shape. We theorized that the angle of repose would increase steadily with particle aspect ratio; as a consequence, the runoff distance would smoothly go to zero as the pile became more solid-like.

True enough, and as expected, piles made from rods with a large aspect ratio did not collapse. But basically, we could not have been more wrong. When a pile did collapse, the runoff distance was roughly independent of aspect ratio. The piles spread out as far as ordinary sand would. What truly surprised us, though, was that for a range of aspect ratios and initial pile heights, the pile collapse was probabilistic: Sometimes the pile collapsed like ordinary sand, and sometimes it held its shape as a solid. For a given aspect ratio, we observed a critical lower pile height below which the pile never collapsed and a critical taller pile height above which it always collapsed. For intermediate heights, the probability for collapse increased linearly.

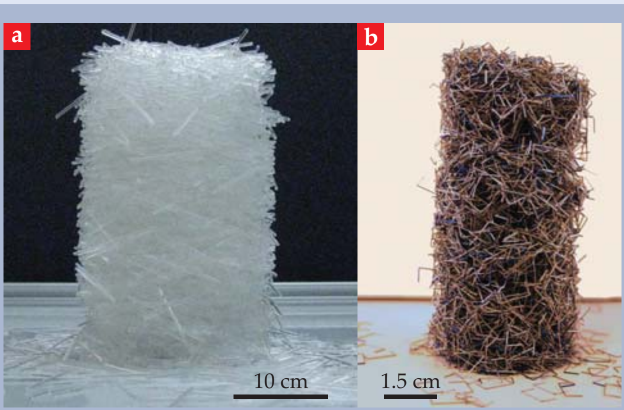

Figure 1. Tall piles of **(a)** long, thin rods and **(b)** staples maintain the shape of the container in which they were initially formed and manifest sharp edges not seen with granular sand.

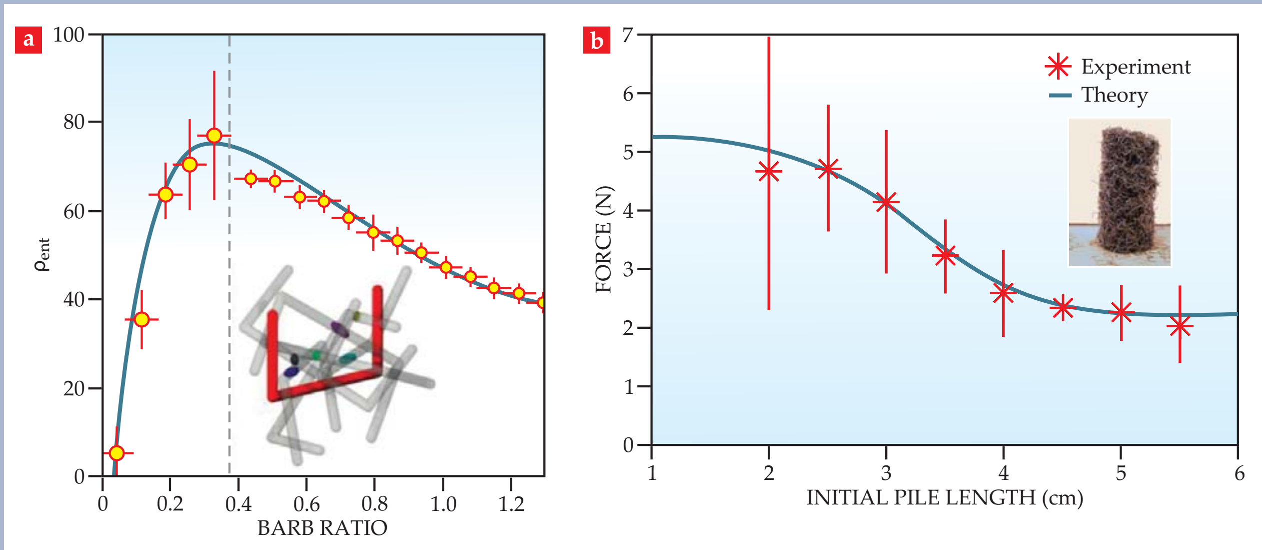

Figure 2. (a) Entangled staples mutually pierce each other's bounding area. In the illustration, colored marks indicate where the red staple's bounding area is pierced by the surrounding gray staples. The product of the average number of entanglements and the packing fraction is the entanglement density ρ_{ent}. Simulations (solid line) indicate that the density peaks at a barb (staple arm to crossbar length) ratio of about 0.4, very nearly the ratio at which experimental piles (circles) are most rigid (dashed line). (Adapted from N. Gravish, S. V. Franklin, D. L. Hu, D. I. Goldman, *Phys. Rev. Lett.* **108**, 208001, 2012.) **(b)** The force needed to pull apart a thin pile of staples (stars) drops dramatically once the pile becomes about 4 cm long. The relatively sudden drop can be explained by positing that long piles be considered as multiples of units with the fundamental length ξ mentioned in the text. Our best theoretical fit to the data (solid curve) yields $\xi = 3.5$.

Pulling and stretching staples

New experiments on U-shaped staples have broadened our conception of geometrically cohesive particles and have suggested approaches to a more comprehensive theory for understanding their behavior. Earlier this year, Nicholas Gravish and Daniel Goldman investigated the stability of staple piles by shaking them and measuring their height h as a function of time t. They found that the height decayed as a stretched exponential, $h(t) \propto \exp[(-t/\tau)^{\beta}]$, a kind of melting that could be interpreted as resulting from activation over an energy barrier during a characteristic time τ. (The exponent β is a dimensionless constant.) The height of the barrier could be estimated by seeing how τ changed as the shaking became more vigorous; in fact, the decay time grew exponentially with the inverse of the shaking acceleration. Subsequent experiments on staples with different barb ratios (the ratio of arm to crossbar length) revealed a surprising nonmonotonic behavior of rigidity: There is an optimum barb ratio at which staples form the most rigid piles.

The key to explaining that nonmonotonic behavior lies in the density of particle entanglements ρ_{ent}. Two staples are said to be entangled if they pierce each other's bounding areas. The idea is illustrated in figure 2a, which shows a number of staples that all pierce the bounding area of a particular one. The product of the average number of entanglements per particle and the packing fraction gives the entanglement density ρ_{ent}.

Increasing the barb ratio at fixed crossbar size increases the bounding area, and so the absolute number of entanglements per particle increases. But as the barb ratio increases, individual staples take up more space—the excluded volume increases—and so the packing fraction goes down. Eventually, the decrease in packing fraction overwhelms the increase in number entanglements. And, as displayed in figure 2a, data from simulations show a peak in ρ_{ent} very near the experimentally observed barb ratio for maximally rigid piles.

In experiments currently under way in my research group, we fix a pile of staples at one end and attach the other to a spring whose free end is pulled at a constant slow velocity. We then monitor both the force applied to the pile and the length of the pile. As expected, the pile grows in bursts as staples disentangle, showing the stick-slip characteristic of motion through a granular material.

Surprisingly, the force needed to pull apart a long, thin pile is much less than for a shorter pile; long piles are weak. This is not an elastic effect in which an effective spring constant scales inversely with length. Rather, as shown in figure 2b, the weakening comes on fairly rapidly near a critical length. We have been working to understand the critical weakening in the context of a weakest-link theory that posits the existence of a fundamental length scale ξ and a distribution of link strengths. Longer piles are composed of multiple units chained together, and so the probability of finding a weak link increases. The approach seems promising, and we are attempting to develop a fully satisfactory theory.

Our understanding of geometric cohesion has evolved in recent years, but the experimental and computational studies needed to complete the picture are only just beginning. Molecular dynamics simulations that accurately model the elastic and frictional forces on each particle will shed light on how particles rearrange when disturbed, how disturbances percolate through the entire pile and lead to global collapse, and how particle motion correlates with interparticle force. Physicists are contemplating new cohesive geometries—for example, systems including arcs with various lengths. Work continues in a practical direction too, and it may lead to rigid objects that are both porous and lightweight.

Additional resources

▶ A. P. Philipse, "The random contact equation and its implications for (colloidal) rods in packings, suspensions, and anisotropic powders," *Langmuir* **12**, 1127 (1996).

▶ K. Desmond, S. V. Franklin, "Jamming of three-dimensional prolate granular materials," *Phys. Rev. E.* **73**, 031306 (2006).

▶ M. Trepanier, S. V. Franklin, "Column collapse of granular rods," *Phys. Rev. E.* **82**, 011308 (2010).