

Name: _____

Final

This is a three hour closed-book exam. Please put your name on the top sheet. Answer all seven questions. Make any necessary assumptions and state them. Explain your working.

Problem	Points
1. (30)	
2. (15)	
3. (15)	
4. (15)	
5. (15)	
6. (15)	
7. (15)	
TOTAL =	

HW = % QUIZZES = % FINAL = %

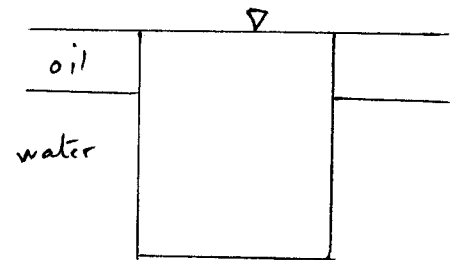
COURSE GRADE =

Exams will be available from staff in EBU II 573 with proper identification, from December 16 on. The final exam will have your course grade on it.

1 (30 points) Answer the following questions *as briefly as possible*.

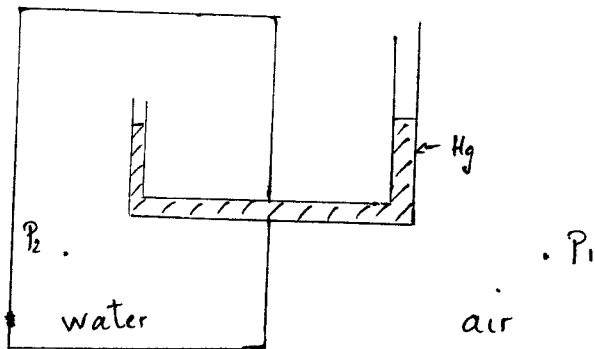
A (5 points) Fluid motion in air is characterized by the streamfunction $\psi_1 = x^3 - 6xy^2$. Determine the velocity field and sketch the streamlines. Is the flow steady? Is the flow irrotational? Compute the components of the shear stress at the point (1, 1).

B (5 points) A cube floats with its top surface level with the surface of a fluid. The upper third of the cube is in oil with specific gravity 0.7 and the lower two thirds in water (see diagram). What is the density of the cube?



F (4 points) Compute the Reynolds number for flow past a hummingbird of length 10 cm flying at 54 km/h. In a water tunnel with a flow speed of 10 m s^{-1} , what size should the model be for dynamical similarity?

G (2 points) Sketched is a manometer system. Fill the blanks with P_1 and P_2 according to which is greater.



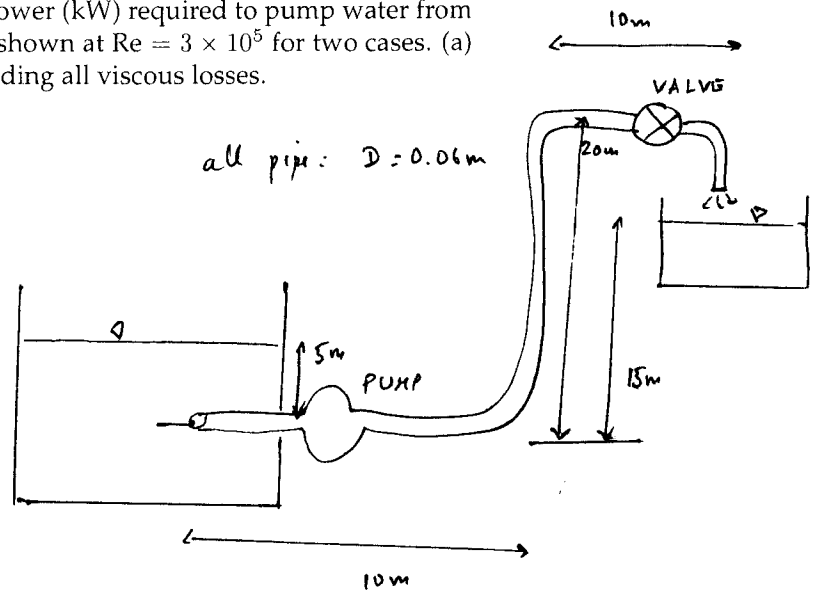
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H (3 points) A 40-mm diameter table tennis ball can travel at 112.5 km/h. Calculate the drag on the ball. [The projected area of a sphere is $\pi d^2/4$.]

2 (15 points) A watertight bulkhead 22 ft high forms a temporary dam for some construction work. The top 12 ft behind the bulkhead consist of seawater with density 2 slugs/ft^3 , with the bottom 10 ft being a mixture of mud and water can be considered a fluid of density 4 slugs/ft^3 . Calculate the total horizontal load per unit width and the location of the center of pressure measured from the bottom.

3 (15 points) Water flows through a straight section of a 5 cm-ID concrete pipe (roughness 2 mm) with an average velocity of 4 m s^{-1} . The pipe is 500 m long, and there is an increase in elevation of 1 m from the inlet of the pipe to its exit. Find the power required to produce this flow rate for the specified conditions.

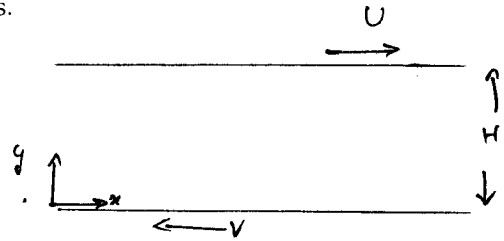
4 (15 points) Determine the pump power (kW) required to pump water from one reservoir to a higher reservoir as shown at $Re = 3 \times 10^5$ for two cases. (a) Neglecting all viscous losses; (b) including all viscous losses.



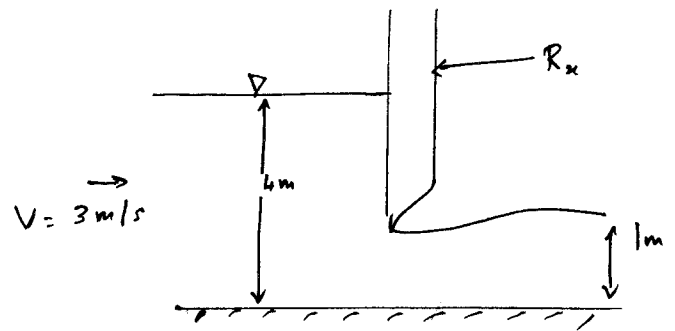
5 (15 points) Consider incompressible, steady, fully-developed flow of a water film of thickness H between two horizontal plates as shown. The upper plate moves to the right with speed U and the lower plate moves to the left with speed V .

(a) Solve for the velocity distribution $U(y)$ as a function of H , U , V , ρ and μ , in the coordinate system shown.

(b) Find V so that the velocity at $y = H/4$ vanishes.



6 (15 points) Water flows under a sluice gate as shown. Neglecting the viscous shear force along the channel bottom, determine the magnitude of the horizontal force per unit width, R_x (N/m), required to hold the gate in place. You must clearly show your control volume and the evaluation of your area integrals to get full credit.



7 (15 points) The viscosity μ of a liquid can be measured by determining the time t it takes for a sphere of diameter d to settle slowly through a distance l in a vertical cylinder of diameter D containing the liquid. Assume that $t = f(l, d, D, \mu, \Delta\gamma)$, where $\Delta\gamma$ is the difference in specific weights between the sphere and the liquid. Use dimensional analysis to show how t is related to μ , and describe how such an apparatus might be used to measure viscosity.

The following *may be useful*:

1 The Navier–Stokes equations are

Mass: $\nabla \cdot \mathbf{V} = 0$.

Momentum: $\rho D\mathbf{V}/Dt = \rho \partial \mathbf{V} / \partial t + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V}$.

Example (*x*-momentum): $\rho \partial u / \partial t + \rho (u \partial u / \partial x + v \partial u / \partial y + w \partial u / \partial z) = \rho g_x - \partial p / \partial x + \mu (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 + \partial^2 u / \partial z^2)$.

Stress tensor: $\sigma_{ij} = -p \delta_{ij} + \mu (\partial V_i / \partial x_j + \partial V_j / \partial x_i)$.

Example: $\tau_{xy} = \mu (\partial v / \partial x + \partial u / \partial y)$ is stress on *x*-face in *y*-direction.

2 The Extended Bernoulli Equation is $\Delta(p/\gamma + V^2/2g + z) = -h_L + h_S$;

$$h_L = f \frac{L V^2}{D 2g}, \quad h_L = K_L \frac{V^2}{2g}, \quad \dot{W}_S = \gamma Q h_S.$$

3 The steady-state CV mass conservation equation is $0 = \int (\rho \mathbf{V} \cdot \mathbf{n}) dA$.

4 The steady-state CV momentum equation is $\sum \mathbf{F} = \int (\rho \mathbf{V})(\mathbf{V} \cdot \mathbf{n}) dA$.

5 Potential and stream function: $u = \partial \phi / \partial x = \partial \psi / \partial y$; $v = \partial \phi / \partial y = -\partial \psi / \partial x$.

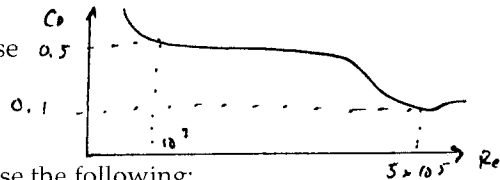
6A For pipe friction factors, use the following:

Laminar friction factors ($Re < 2100$): $f = 64/Re$.

Turbulent smooth wall friction factors: $f = 0.32/Re^{1/4}$.

Completely rough, fully turbulent flow: $f = (\epsilon/D)^{1/2}$.

6B For the drag on spheres, use



7 For minor loss coefficients use the following:

90° elbow = 1.5; 45° elbow = 0.5; 180° return = 1.5; valve = 5.0.

Sudden contraction (based on V_2) = 0.3; sudden expansion (based on V_1) = 0.4; entrance flow = 0.5; exit flow = 1.0.

8 For fluid properties, use the following:

Fluid	ρ (kg/m ³)	μ (Ns/m ²)	ν (m ² /s)	σ (surface tension, N/m)
Water	1000	0.001	10^{-6}	0.1
Air	1.0	2×10^{-5}	2×10^{-5}	

$g = 9.8 \text{ m/s}^2$; 1 atm = $10^5 \text{ Pa} = 10^5 \text{ N/m}^2$.