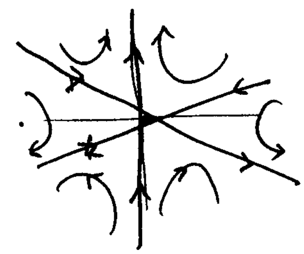
1 (30 points) Answer the following questions *as briefly as possible.*

$x = 0$
$x^2 = 6y^2$

A (5 points) Fluid motion in air is characterized by the streamfunction $\psi_1 = x^3 - 6xy^2$. Determine the velocity field and sketch the streamlines. Is the flow steady? Is the flow irrotational? Compute the components of the shear stress at the point $(1, 1)$.



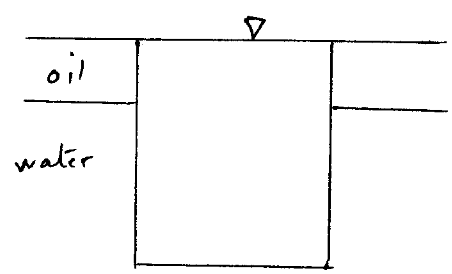
$$\underline{u} = (\psi_y, -\psi_x) = (-12xy, -3x^2 + 6y^2)$$

Steady

$$\nabla^2\psi = 6x - 12x \neq 0 \qquad \underline{\text{not}} \text{ irrotational}$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left(-12x - 6x\right)$$

B (5 points) A cube floats with its top surface level with the surface of a fluid. The upper third of the cube is in oil with specific gravity 0.7 and the lower two thirds in water (see diagram). What is the density of the cube?



Archimedes: $Mg = \rho_{oil} V_{oil} g + \rho_w V_w g$

$M = \rho V, \quad V_{oil} = \frac{1}{3} V, \quad V_w = \frac{2}{3} V$

$\Rightarrow \quad \rho = \frac{1}{3}\rho_{oil} + \frac{2}{3}\rho_w = \left[\frac{1}{3} \times 0.7 + \frac{2}{3}\right]\rho_w$

$= \frac{2.7}{3}\rho_w = 0.9\rho_w = 900 \text{ kg m}^{-3}$

C (3 points) Define the following terms: (a) *Newtonian fluid,* (b) *control volume,* (c) *minor loss.*

(a) Shear stress proportional to rate of strain

(b) Geometric volume drawn in fluid ; fluid can flow in or out.

(c) Head loss due to fittings

D (4 points) A 10-cm diameter cylinder filled with water is in solid body rotation. What is the pressure difference between the axis of rotation and the edge of the cylinder? Where is the pressure higher?

$$-\frac{v^2}{r} = -\Omega^2 r = -\frac{1}{\rho}\frac{\partial p}{\partial r} \Rightarrow p = p_0 + \frac{1}{2}\rho\Omega^2 r^2$$

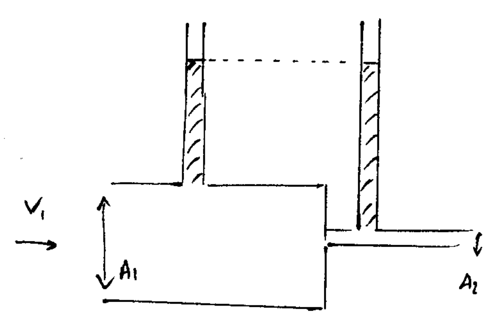
$$p_b = p_0 + \frac{1}{2}\rho\Omega^2\left(\frac{d}{2}\right)^2$$ pressure higher at boundary

$$\Delta p = 1000 \times \frac{1}{2} \times \Omega^2 \times (0.05)^2 = 125\ \Omega^2 \quad (\Omega \text{ in } s^{-1})$$

E (4 points) Two *static* pressure taps are located just upstream and downstream, respectively, of a **sudden contraction**, as shown. What contraction area (A_1/A_2) will cause the manometer heights to be equal?

EB: $z_1 = z_2$, $p_1 = p_2 = p_{atm}$
[HS in 3-dim]

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} - h_L$$



Mass conservation: $V_1 A_1 = V_2 A_2$

Minor loss due to expansion $h_L = 0.3\frac{V_2^2}{2g}$

$$\Rightarrow V_2^2 = V_1^2 - 0.3V_2^2$$

$$\frac{A_1}{A_2} = \frac{V_2}{V_1} = \frac{1}{\sqrt{1.3}}$$

contradiction (mistake in qn)

F (4 points) Compute the Reynolds number for flow past a hummingbird of length 10 cm flying at at 54 km/h. In a water tunnel with a flow speed of 10 m s^{-1}, what size should the model be for dynamical similarity?

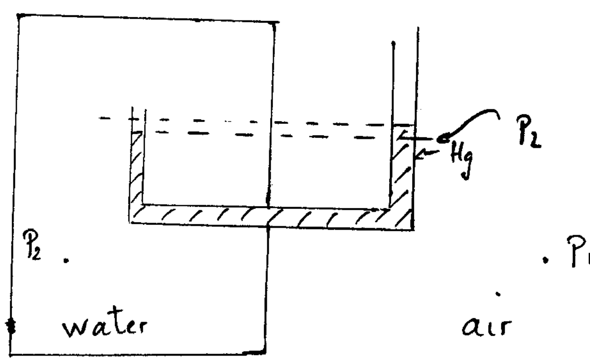
$1 \text{ m s}^{-1} = 3.6 \text{ km h}^{-1}$

$$Re = \frac{(54/3.6) \times 0.1}{2 \times 10^{-5}} = 75\,000$$

Dynamical similarity : $Re_p = Re_m = 75\,000 = \dfrac{10 \times L}{10^{-6}}$

$L = 10^{-7} \times 75\,000 = 7.5 \text{ mm}$

G (2 points) Sketched is a manometer system. Fill the blanks with P_1 and P_2 according to which is greater.



$\underline{P_2} > \underline{P_1}$

In Hg column : P_1 above
$P_2 > P_1$!

H (3 points) A 40-mm diameter table tennis ball can travel at 112.5 km/h. Calculate the drag on the ball. [The projected area of a sphere is $\pi d^2/4$.]

$$F = \frac{1}{2}\rho U^2 A_p C_D = \frac{1}{2}\rho U^2 \frac{\pi d^2}{4} C_D$$

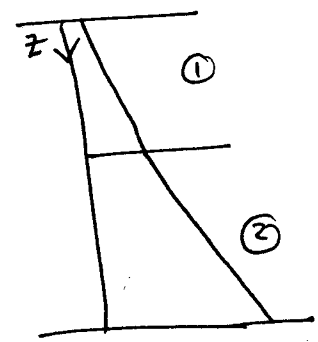
$$Re = \frac{Ud}{\nu} = \frac{(112.5/3.6) \times 0.04}{2 \times 10^{-5}} = 62\,500$$

From curve, $C_D \sim 0.5$

$$F = \frac{1}{2} \times 1.0 \times \left(\frac{112.5}{3.6}\right)^2 \times \frac{\pi}{4} \times (0.04)^2 \times 0.5$$

$$= 0.307 \text{ N}$$

2 (15 points) A watertight bulkhead 22 ft high forms a temporary dam for some construction work. The top 12 ft behind the bulkhead consist of seawater with density 2 slugs/ft^3, with the bottom 10 ft being a mixture of mud and water can be considered a fluid of density 4 slugs/ft^3. Calculate the total horizontal load per unit width and the location of the center of pressure measured from the bottom.



$$p_{1G} = \rho_w g z$$

$$p_{2G} = \rho_m g (z-12) + \rho_w g\, 12$$

$$F = \int_0^{12} \rho_w g z \, dz + \int_{12}^{22} [\rho_m g (z-12) + \rho_w g \, 12] \, dz$$

$$= \rho_w g (72 + 120) \text{ ft}^2 + \rho_m g \, 50 \text{ ft}^2 = 18\,790 \text{ lbf}$$

Moment about top of wall:

$$M = \int_0^{12} z \rho_w g z \, dz + \int_{12}^{22} z [\rho_m g (z-12) + \rho_w g 12] \, dz$$

$$= 288,450 \text{ ft lbf}$$

Center of pressure 15.35 ft below top of wall

i.e. 6.65 ft above base

3 (15 points) Water flows through a straight section of a 5 cm-ID concrete pipe (roughness 2 mm) with an average velocity of 4 m s^{-1}. The pipe is 500 m long, and there is an increase in elevation of 1 m from the inlet of the pipe to its exit. Find the power required to produce this flow rate for the specified conditions.

$$Re = \frac{UD}{\nu} = \frac{4 \times 0.05}{10^{-6}} = 2 \times 10^{5}$$

Completely rough + turbulent $f = \left(\frac{\varepsilon}{D}\right)^{\frac{1}{2}} = \left(\frac{2 \times 10^{-3}}{5 \times 10^{-2}}\right)^{\frac{1}{2}}$

$$= \sqrt{0.04} = 0.2$$

EB ; p & V same so $\Delta z = -h_L + h_s$

$$\rightarrow h_s = \Delta z + f \frac{L}{D} \frac{V^2}{2g}$$

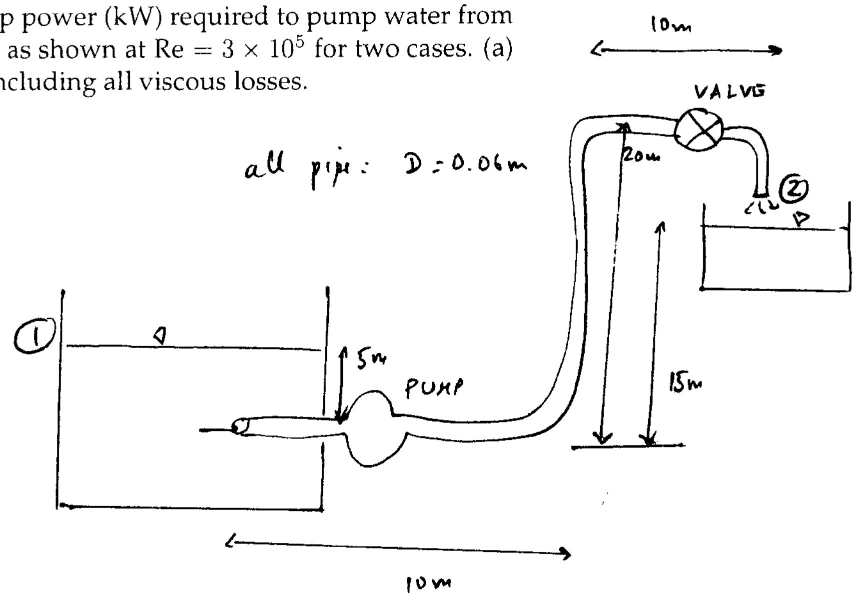
$$= 1 + 0.2 \times \frac{500}{0.05} \times \frac{4^2}{2 \times 9.81}$$

$$= 1.632 \times 10^{3} \text{ m}$$

$$W = \dot{m} g h_s = 4 \times 1000 \times \frac{\pi}{4} \times (0.05)^2 \times 9.81 \times 1.632 \times 10^{3}$$

$$= 125.74 \text{ kW!}$$

4 (15 points) Determine the pump power (kW) required to pump water from one reservoir to a higher reservoir as shown at Re = 3×10^5 for two cases. (a) Neglecting all viscous losses; (b) including all viscous losses.



$$Re = \frac{UD}{\nu} = 3 \times 10^5$$

$$= \frac{U \times 0.06}{10^{-6}}$$

$$\Rightarrow U = 5 \text{ m s}^{-1}$$

$$Q = \frac{\pi D^2}{4} = 0.014 \text{ m}^3 \text{s}^{-1}$$

(a) Pump just overcomes gravity + vel. changes

[big reservoir at 1: $V_1 = 0$]

$$\frac{V_2^2}{2g} + z_2 = z_1 + h_s \qquad h_s = \frac{5^2}{2 \times 9.8} + 15 = 16.28\text{m}$$

$$\dot{W} = \rho g h_s Q = 9.8 \times 1000 \times 16.28 \times 0.014 = 2.255 \text{ kW}$$

(b) Add losses:

$$\sum K_L = \underset{\text{entry}}{0.5} + \underset{\text{elbow}}{3 \times 1.5} + \underset{\text{valve}}{5}$$

$$f = \frac{0.032}{Re^{1/4}} = 0.0137$$

$$\text{so} \quad h_L = \left[0.5 + 3 \times 1.5 + 5 + 0.037 \times \frac{40}{0.06}\right] = 24.4\text{m}$$

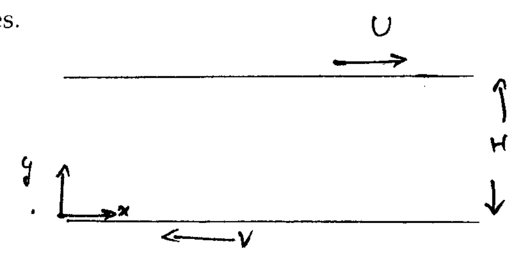
$$h_s = 16.28 + 24.4 = 40.66\text{m}$$

$$\rightarrow P = \rho g h_s Q = 5.1 \text{ kW}$$

5 (15 points) Consider incompressible, steady, fully-developed flow of a water film of thickness H between two horizontal plates as shown. The upper plate moves to the right with speed U and the lower plate moves to the left with speed V.

(a) Solve for the velocity distribution $U(y)$ as a function of H, U, V, ρ and μ, *in the coordinate system shown.*

(b) Find V so that the velocity at $y = H/4$ vanishes.



Fully-develoed, steady

no pressure gradient.

(a) NS x :

$$0 = \mu \frac{d^2 u}{dy^2} \Rightarrow u = Ay + B$$

BCs :

$$\left.\begin{array}{ll} u = -V & y = 0 \\ u = U & y = H \end{array}\right\} \quad u = (U+V)\frac{y}{H} - V$$

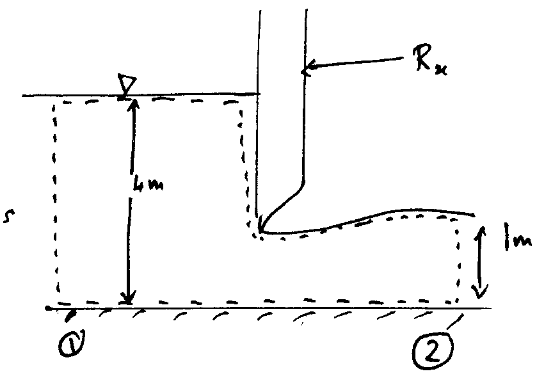
(b)

$$u(y = H/4) = (U+V)\frac{1}{4} - V$$

$$u(y = H/4) = 0 \Leftrightarrow \frac{(U+V)}{4} - V = 0$$

$$\Leftrightarrow V = \frac{U}{3}$$

6 (15 points) Water flows under a sluice gate as shown. Neglecting the viscous shear force along the channel bottom, determine the magnitude of the horizontal force per unit width, R_x (N/m), required to hold the gate in place. You must clearly show your control volume and the evaluation of your area integrals to get full credit.



Steady-state momentum

$$\sum F_x = \int \rho \underline{V} (\underline{V} \cdot \underline{n})\, dA$$

$dA = w\,dy$ (w: width)

x-cpt

Pressure force ①: $\frac{1}{2}\rho g H_1^2 w$ (to right)

② $-\frac{1}{2}\rho g H_2^2 w$ (to left)

Flux ①: $\int \rho V_x (-V_x)\, dA = -\rho V_1^2 w H_1$

② $\int \rho V_x (V_x)\, dA = \rho V_2^2 w H_2$

$$\Rightarrow \frac{1}{2}\rho g H_1^2 - \frac{1}{2}\rho g H_2^2 + R_x = -\rho V_1^2 H_1 + \rho V_2^2 H_2$$

+ sign since this is force on fluid = − force on gate!

Conservation of mass: unknown $V_2 = V_1 \frac{H_1}{H_2}$

$$\Rightarrow R_x = -\rho \left\{ \frac{1}{2} g (H_1^2 - H_2^2) + \underbrace{(V_1 - V_2) V_1 H_1}_{V_1^2 H_1 \left(1 - \frac{H_1}{H_2}\right)} \right\}$$

$= 34.425$ kN

7 (15 points) The viscosity μ of a liquid can be measured by determining the time t it takes for a sphere of diameter d to settle slowly through a distance l in a vertical cylinder of diameter D containing the liquid. Assume that $t = f(l, d, D, \mu, \Delta\gamma)$, where $\Delta\gamma$ is the difference in specific weights between the sphere and the liquid. Use dimensional analysis to show how t is related to μ, and describe how such an apparatus might be used to measure viscosity.

Buckingham Π θ^m:

t	l	d	D	μ	$\Delta\gamma$
T	L	L	L	$ML^{-1}T^{-1}$	$ML^{-2}T^{-2}$

[NB: specific gravity $\sim \rho g \sim \frac{kg}{m^3} \times \frac{m}{s^2}$]

3 units $\rightarrow$ 3 nondimensional parameters

$$\pi_1 = \frac{l}{D} \qquad \pi_2 = \frac{d}{D} \qquad \pi_3 = \frac{D t \Delta\gamma}{\mu}$$

$$\Rightarrow \quad t = \frac{\mu}{D \Delta\gamma} \phi\left(\frac{l}{D}, \frac{d}{D}\right)$$

Use apparatus to compute π_3 for known fluid.

Use same apparatus with unknown fluid and measure t. Then knowing t and π_3 gives μ.

NB: same apparatus means π_1 and π_2 are same.