Homework X.

Due *Friday December 1, 2006*, in fourth hour.

Read: Chapter 14

Problems:

1 For fully developed flow of glycerin in a smooth 0.2 m pipe at a rate of 5 m^3 s^{-1}, determine the thickness of the laminar sublayer, buffer layer and turbulent core.

2–5 14.6, 14.8, 14.15, 14.18

Comments:

The Navier–Stokes equations describe incompressible turbulent flow, but they are very difficult to solve. We will only attempt to solve for time-averaged flow properties, which introduces the Reynolds stress term. The Navier–Stokes equations can be combined with the continuity equation to give (in suffix form)

$$\frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_j u_k}{\partial x_k} = \rho g_j - \frac{\partial p}{\partial x_j} + \frac{\partial \sigma_{ij}}{\partial x_j}.$$

We write the velocity is a time-average plus a perturbation, i.e. $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$, and similarly for pressure. Substituting into the equations of motion and averaging gives what appears to be the usual Navier–Stokes equations for the average velocity, with the presence of an addition **Reynolds stress** term $\tau_R = \partial_k(\overline{\rho u_j' u_k'})$ – a time correlation between velocity fluctuations. This gives a modified stress tensor $\sigma_{ij} - \rho\overline{\rho u_i' u_j'}$. One model for the turbulent stress tensor is (13-13) where the mixing length is a new parameter. Experimentally, $\tau_R \gg \tau_{ij}$, except near solid surfaces. This leads to flatter velocity profiles and higher head losses and friction factors.

Chapter 14 is a very important engineering chapter. It shows how to carry out flow and pumping calculations through pipes and ducts using a variety of engineering head correlations. You must appreciate the role of the Reynolds number $Re \equiv \rho V D/\mu$. Re determines which regime the flow is in (laminar or turbulent), and appears in most correlations for pressure drop, head loss, friction factor, drag, and so on. Review the text's examples. It is important that you understand the Moody chart in figure 14.1. You should also know what is meant by "neglect minor losses": this refers to piping components.

HAPPY THANKSGIVING (2)