

Homework II.

Due *Friday October 6, 2006*, in fourth hour.

Read: Chapters 3 and 4

Problems:

1. For the velocity field $\mathbf{u} = (x, yx)$, plot the streamline that passes through the point $(1, 1)$. Without doing any further calculations, find the particle path of a particle released at $(2, e)$.
2. Water flows between two plates at $y = 0, h$ with a velocity profile $u = Uy^n(h - y)$. Find n such that the average velocity is $hU/3$. [Hint: you will find two solutions. Explain why only one of them is physically relevant.]
3. The wind blows directly through a $7 \text{ ft} \times 8 \text{ ft}$ garage door with a speed of 1 ft/s . Determine the average speed of the air through two $2 \text{ ft} \times 3 \text{ ft}$ windows. Repeat the calculation if the wind is oriented at 45° degrees to the door.
4. A 10 litre container is initially filled with water. 80 degrees proof alcohol (40% by volume) enters the tank at a rate of 0.2 l/min . The solution is kept uniform by stirring, and the well-mixed liquid flows out to keep the volume constant. How long before the tank is 20% proof?
5. A spherical vessels of radius R is pressurized by filling it with air of density ρ_1 and incoming velocity v through an orifice of radius a . Calculate the pressure in the vessel as a function of time.

Quiz I: The first quiz will be on Friday October 6. The exam will be closed book and will cover the material from chapters 1 and 2.

Comments:

Chapter 3 is concerned with the *kinematics* or motion of fluid. Streamlines are parallel to the flow at any instant. Streaklines are the trajectories of a tracer (smoke, dye) released at a fixed point. Pathlines are the trajectories of particles. The three are **different** in general, but coincide for **steady** flow.

The material derivative D/Dt corresponds to riding with a particle as it moves. This is different from the partial derivative $\partial/\partial t$, which corresponds to sitting at a fixed point in space. Even when the flow is steady, which means that $\partial/\partial t = 0$, the value of a quantity can change as we move with the particle. For example acceleration becomes $\partial\mathbf{v}/\partial t + (\mathbf{v} \cdot \nabla)\mathbf{v}$ (Chapter 9). Make

sure you know how to use vector operators so that you can compute material derivatives.

We will use the Eulerian description of fluid flow: velocity and other quantities are described as functions of space and time. We look at what happens at fixed points, or in fixed Control Volumes (CV). Fluid can flow through the boundaries of a CV. The equations of physics however refer to well-defined systems – Control Systems (CS) in our parlance. A CS always contains the same material and hence fluid cannot pass through its boundaries. The relation of Chapter 4, Section 1, is the Reynolds Transport Theorem that allows us to go from CV to CS and work in the Eulerian framework.

Chapter 4 starts to introduce the conservation equations appropriate to macroscopic control volumes that are used to analyze systems. The Reynolds Transport Theorem is applied to obtain equations for the conservation of mass (4-1); momentum and energy come in subsequent chapters.