CENG101A: Introductory Fluid Mechanics Fall Quarter 2006
http://maecourses.ucsd.edu/mae210a

Homework IX.

Due *Monday November 27, 2006.*

Read: Chapter 13

Problems:

1. What is the drag on the Sears Tower in Chicago on a windy day? Explain your estimates and assumptions.
2. Write a paragraph on the drag on swimming animals, ranging from microorganisms to whales. What regimes occur? How is the shape of the animals adapted to these regimes? [Cite references.]
3. Carry out the momentum integral analysis for the case of no pressure gradient with $v_x = a \sin by$ (you will have to solve for a and b.) Compare with the exact Blasius result.
4. Compute the momentum and displacement thicknesses for the previous problem.

Comments:

We did Chapter 12 this week. The special problem is an unsteady solution to the Navier–Stokes equations that requires a similarity variable. It is a model problem for boundary layers.

Chapter 12 deals with boundary layers: thin viscous regions near surfaces in which viscous shear effects occur. Outside boundary layers, the flow is like an inviscid potential flow. You will be expected to follow the Blasius solution, which uses a similarity solution to solve the boundary layer problem (in a similar fashion to the homework problem), and to know how to use the integral momentum boundary layer analysis to compute the boundary layer thickness and drag on flat plates. You will need to be aware of the transition to turbulent boundary flow and how it alters the friction drag

Two important flow you should review are (1) flow in a channel in which an adverse pressure gradient can cause a backward flow even when the flow is being dragged forward by a shear force; and (2) the potential flow around a cylinder of section 10.3, which shows how both negative and positive pressure gradients occur due to body shape, and that without viscosity there is zero drag.

Special Problem: The Impulsive Plate

Consider a semi-infinite flat plate at $y = 0$, with an infinite static incompressible fluid in the half-plane $y > 0$. At $t = 0$, the plate is impulsively given a constant velocity $u = U_p$ in the x-direction. As a function of time and height above the plate, the fluid will begin to move, coupled to the plate by viscous shear forces. Solve for the resulting fluid velocity profile $u(t, y)$, the shear stress at the plate $\tau_w = f(t, \mu, \rho, U_p)$, and find the shear layer thickness δ, following the steps below.

1 Since the plate is infinite and moves only in the x-direction, take $\partial_y = \partial_z = 0$ and $v = w = 0$. Show that Navier–Stokes equations reduce to

$$\frac{\partial u}{\partial t} = \nu \frac{\partial u}{\partial y^2}.$$

This is a parabolic, one-dimensional (in space) diffusion equation with diffusion constant ν. Show that the appropriate boundary conditions are: $u = 0$ for $t = 0$ for all y; and $u = 0$ as $y \to \infty$ and $u = U_p$ at $y = 0$ for $t > 0$.

2 Define the dimensionless similarity variable $\eta \equiv y/\sqrt{4\nu t}$, and assume that $u/U_p = f(\eta)$ is a function only of η (i.e. $\partial f/\partial\eta = \mathrm{d}f/\mathrm{d}\eta$). Using the chain rule, show that the PDE and its boundary conditions combine to give the ODE

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} + 2\eta\frac{\mathrm{d}f}{\mathrm{d}\eta} = 0 \qquad \text{with } f = 1 \text{ at } \eta = 0 \text{ and } f = 0 \text{ as } \eta \to \infty.$$

3 Integrate the ODE directly twice together with the boundary conditions to get $f = u/U_p = \mathrm{erfc}(\eta) = 1 - \mathrm{erf}(\eta)$. The complementary error function is given by $\mathrm{erfc}(\eta) = 1 - (2/\sqrt{\pi}) \int_0^\eta \mathrm{e}^{-u^2}\, \mathrm{d}u$. You will want to know $\int_0^\infty \mathrm{e}^{-x^2}\, \mathrm{d}x = \sqrt{\pi}/2$; $\mathrm{erfc}(1.82) = 0/01$.

4 Using the solution for u/U_p, find $\tau_w = \mu\partial u/\partial y|_{y=0}$ as a function of U_p, t, μ, ρ. Show that this solution is similar to (12-30) except for the coefficient (for the flat plate, time is related to flow down the plate by $y = x/U$.)

5 A traditional estimate of the viscous shear layer thickness is the distance $y = \delta$ where viscous effects have dropped to 1% of wall values, or for this problem, $u/U_p = f(\eta) = 0.01$. Show that $\delta = 3.64\sqrt{\nu t}$, which indicates how viscous effects diffuse or penetrate into the fluid. Again, using $t = x/U$, compare with the flat plate result (12-28).

HAPPY THANKSGIVING (1)