CENG101A: Introductory Fluid Mechanics Fall Quarter 2006
http://maecourses.ucsd.edu/mae210a

Quiz I Solution

Joseph Lagrange (1736–1813)

1

1. A fluid is a substance that deforms continuously when subjected to a shear stress.
2. Dimensions should be the same on both sides of an equation.
3. Pressure is minus the normal stress in a fluid at rest.

2

$$\begin{aligned}
\mathbf{a}\cdot\mathbf{b} &= (3yz-2x^3)+2y(-1)+z^2(2x^2),\\
\mathbf{a}\cdot\nabla f &= (3yz-2x^3)(5-2yz-2z^2)+2y(2xz)+z^2(2xy+4xz),\\
\nabla\cdot\mathbf{a} &= -6x^2+2+2z,\\
\nabla\times\mathbf{b} &= (0,-4xz,1),\\
\nabla(f\mathbf{u}) & \quad \text{does not make sense.}
\end{aligned}$$

3 Archimedes: the weight of the iceberg is $\rho_i V$. The weight of the displaced water is $\rho_w V_s$. The two are equal since the iceberg floats, so $V_s/V = \rho_i/\rho_w = 925/1030 = 89.8\%$. This is independent of the shape of the iceberg. The center of pressure is below the centroid of the iceberg (the pressure is greater at the base of the iceberg so when equating moments the center of pressure must be below the centroid). In fact the center of pressure is below the centroid of the submerged portion of the iceberg, which is itself below the centroid of the whole iceberg. If the water is warmer, ρ_w decreases. Assuming that ρ_i does not change, V_s/V must increase, so more of the iceberg is submerged (this ignores complications caused by the strange density–temperature relation of water near its freezing point).

4 Measure z from the top. The gage pressure in the upper layer is $p_{1G} = \rho_w g z$. The gage pressure in the lower layer is $p_{2G} = \rho_m g(z - 10) + \rho_w g 10$. Integrate to get the force per unit width:

$$\begin{aligned} F &= \int_0^{10} \rho_w g z \, \mathrm{d}z + \int_{10}^{18} [\rho_m g(z - 10) + \rho_w g 10] \, \mathrm{d}z \\ &= \rho_w g(50 + 80)\mathrm{ft}^2 + \rho_m g \, 30\mathrm{ft}^2 = 17,700 \, \mathrm{lb_f}. \end{aligned}$$

To find the center of pressure, compute the moment about the top of the wall:

$$\begin{aligned} M &= \int_0^{10} z \rho_w g z \, \mathrm{d}z + \int_{10}^{18} z[\rho_m g(z - 10) + \rho_w g 10] \, \mathrm{d}z \\ &= \rho_w g(333 + 1120)\mathrm{ft}^2 + \rho_m g \, 490.7\mathrm{ft}^2 = 219,200 \, \mathrm{ft \; lb_f}. \end{aligned}$$

Hence the center of pressure is 12.39 ft below the top of the wall and 5.61 ft above the base of the wall.