

Name: \_\_\_\_\_



## Quiz II

Sir Isaac Newton (1643–1727)

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. Streamlines

- are the same as particle paths.
- are everywhere parallel to the velocity field.
- only exist in two-dimensional flows.
- can be obtained by solving algebraic equations.
- can never form closed loops.

2.  $\nabla \cdot \mathbf{x} =$

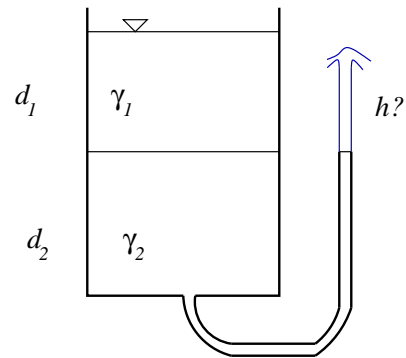
- 3 (in three dimensions).
- is a vector.
- depends on the position of the origin.
- makes no sense.
- points from the origin to the point  $\mathbf{x}$ .

3. The first law of thermodynamics

- is an integral of Newton's Second Law.
- concerns entropy.
- says that energy is always lost.
- never has any fluxes of energy across a control surface.
- governs the energy of a system.

2 (5 points) You are given the two-dimensional velocity field  $\mathbf{u} = (-y, 2x, 0)$ . Compute its divergence and curl. Compute the streamline passing through the point  $(1, 0, 0)$ . Now compute the change in the function  $\psi \equiv 2x^2 + y^2$  along the streamline. Compute  $\nabla^2\psi$ . What does this remind you of?

3 (10 points) A small tube of area  $0.01 \text{ m}^2$  is inserted at the bottom of large tank below a  $d_2 = 2 \text{ m}$  layer of fluid 2 of specific weight  $\gamma_2 = 12 \text{ kN m}^{-3}$ , which itself is below a  $d_1 = 2 \text{ m}$  layer of fluid with  $\gamma_1 = 6 \text{ kN m}^{-3}$ . The open end of the tube is bent up to a height equal to the interface between the two fluids. Neglecting viscous effects, find (a) the velocity of fluid A as it exists the pipe; (b) the height to which the fluid jet will rise above the end of the pipe; (c) the momentum flux through the end of the tube.



4 (12 points) A high-speed channel flow  $(V_1, h_1)$  may “jump” to a low-speed condition  $(V_2, h_2)$ . The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to obtain

$$h_2 = \frac{h_1}{2} \left[ \left( 1 + \frac{8V_1^2}{gh_1} \right)^{1/2} - 1 \right].$$

Show that Bernoulli’s equation applied between sections 1 and 2 does not give this result. Derive an expression for the change in total head across the jump.