CENG101A: Introductory Fluid Mechanics Fall Quarter 2006
http://maecourses.ucsd.edu/mae210a

Name: ____

Quiz II

Daniel Bernoulli (1700–1782)

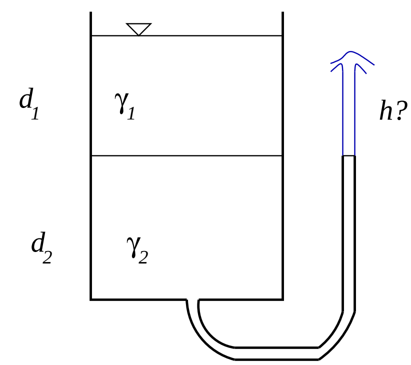
This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. Streamlines
   - can never form closed loops.
   - are everywhere parallel to the velocity field.
   - are the same as particle paths.
   - only exist in two-dimensional flows.
   - can be obtained by solving algebraic equations.
2. $\nabla\cdot\mathbf{x}$ =
   - is a vector.
   - 3 (in three dimensions).
   - points from the origin to the point $\mathbf{x}$.
   - depends on the position of the origin.
   - makes no sense.
3. The first law of thermodynamics
   - concerns entropy.
   - is an integral of Newton's Second Law.
   - never has any fluxes of energy across a control surface.
   - says that energy is always lost.
   - governs the energy of a system.

2 (5 points) You are given the two-dimensional velocity field $\mathbf{u} = (-y, 3x, 0)$. Compute its divergence and curl. Compute the streamline passing through the point $(0, 1, 0)$. Now compute the change in the function $\psi \equiv 3x^2 + y^2$ along the streamline. Compute $\nabla^2\psi$. What does this remind you of?

3 (10 points) A small tube of area 0.02 m^2 is inserted at the bottom of large tank below a $d_2 = 2$ m layer of fluid 2 of specific weight $\gamma_2 = 8$ kN m^{-3}, which itself is below a $d_1 = 2$ m layer of fluid with $\gamma_1 = 4$ kN m^{-3}. The open end of the tube is bent up to a height equal to the interface between the two fluids. Neglecting viscous effects, find (a) the velocity of fluid A as it exists the pipe; (b) the height to which the fluid jet will rise above the end of the pipe; (c) the momentum flux through the end of the tube.



4 (12 points) A high-speed channel flow (V_1, h_1) may "jump" to a low-speed condition (V_2, h_2). The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to obtain

$$h_2 = \frac{h_1}{2}\left[\left(1 + \frac{8V_1^2}{gh_1}\right)^{1/2} - 1\right].$$

Show that Bernoulli's equation applied between sections 1 and 2 does not give this result. Derive an expression for the change in total head across the jump.