

Name: _____



Quiz III

Charles Louis Marie Henri Navier (1785–1836)

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. Newtonian fluids

- have a yield stress.
- have a linear shear stress-rate of shear strain relation.
- are the same as Bingham fluids.
- are inviscid.
- only exist in one dimension.

2. Fully-developed flow

- is only possible for Newtonian fluids.
- means that the velocity profile is linear.
- is only possible for compressible flows.
- requires a free surface.
- does not depend on the coordinate along the pipe (or channel).

3. The Navier–Stokes equation

- is the same as the Euler equation
- was derived by Newton.
- is linear in velocity.
- is derived from Newton's Second Law.
- is an energy equation.

2 (5 points) Two-dimensional fully-developed flow down a channel is given by $\mathbf{u} = (y - y^2, 0)$; the pressure field is $p = y$. The boundaries are at $z = 0$ and $z = 1$. Find the vorticity and shear stress. Write down the Navier-Stokes equation and deduce the velocities of the boundaries and the pressure gradient. In what direction does gravity act?

3 (10 points) Fluid flows between two parallel plates separated by a distance h : the lower is at rest, the upper has velocity U , and there is a constant background pressure gradient dP/dx . Find the relation between U and dP/dx so that the average velocity is zero.

4 (12 points) Derive the Hagen–Poiseuille relation between the pressure drop along a length of pipe and the volume flux, stating carefully your assumptions.