

Name: \_\_\_\_\_



## Quiz III

George Gabriel Stokes (1819–1903)

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. Newtonian fluids

- have a yield stress.
- have a linear shear stress-rate of shear strain relation.
- are the same as Bingham fluids.
- are inviscid.
- only exist in one dimension.

2. Fully-developed flow

- is only possible for Newtonian fluids.
- means that the velocity profile is linear.
- is only possible for compressible flows.
- requires a free surface.
- does not depend on the coordinate along the pipe (or channel).

3. The Navier–Stokes equation

- is the same as the Euler equation
- was derived by Newton.
- is linear in velocity.
- is derived from Newton's Second Law.
- is an energy equation.

2 (5 points) Two-dimensional fully-developed flow down a channel is given by  $\mathbf{u} = (2y - y^2, 0)$ ; the pressure field is  $p = -y$ . The boundaries are at  $z = -1$  and  $z = 0$ . Find the vorticity and shear stress. Write down the Navier-Stokes equation and deduce the velocities of the boundaries and the pressure gradient. In what direction does gravity act?

3 (10 points) Fluid flows between two parallel plates separated by a distance  $h$ : the lower is at rest, the upper has velocity  $U$ , and there is a constant background pressure gradient  $dP/dx$ . Find the relation between  $U$  and  $dP/dx$  so that the average velocity is zero.

4 (12 points) Derive the Hagen–Poiseuille relation between the pressure drop along a length of pipe and the volume flux, stating carefully your assumptions.