

Name: \_\_\_\_\_



## Quiz IV

Pierre-Simon Laplace (1749–1827)

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

- 1 (3 points) Circle the correct answer.
  1. The velocity potential
    - exists for all flows.
    - exists only for incompressible flows.
    - exists only for Newtonian fluids.
    - exists only for irrotational flows.
    - always satisfies Laplace's equation.
  2. The streamfunction
    - exists for all flows.
    - exists only for incompressible flows.
    - exists only for Newtonian fluids.
    - exists only for irrotational flows.
    - always satisfies Laplace's equation.
  3. The Buckingham Pi theorem
    - shows that  $\pi$  is a transcendental number.
    - applies only to irrotational flows.
    - generates dimensionless parameters.
    - is derived from Newton's Second Law.
    - is an energy equation.

2 (5 points) Sketch the streamlines for the flow with streamfunction  $\psi = 3x^2 - y$ . Can you find a velocity potential for this flow?

3 (10 points) The pressure rise  $\Delta p$  across a pump can be expressed as  $\Delta p = f(D, \rho, \omega, Q)$ , where  $D$  is the impeller diameter,  $\rho$  the fluid density,  $\omega$  the rotational speed and  $Q$  the flowrate. Determine a suitable set of dimensionless parameters.

4 (12 points) Derive the solution for flow past a cylinder in two-dimensional incompressible irrotational flow. Show that the drag on the cylinder is zero. [Explain your working clearly. You may use without proof the fact that

$$r \sin \theta \left( 1 - \frac{a^2}{r^2} \right)$$

is a solution of Laplace's equation.]