

Name: _____



Quiz V

Ludwig Prandtl (1875–1953)

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all four questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. In a boundary layer,
 - viscosity is negligible.
 - pressure vanishes at the wall.
 - there is no pressure gradient normal to the boundary.
 - the Reynolds number is always small.
 - velocity satisfies Laplace's equation.
2. The Reynolds stress
 - is the turbulent contribution to shear stress.
 - depends on mean velocity gradients.
 - appears in the mean continuity equation.
 - vanishes in two-dimensional flows.
 - is proportional to the Reynolds number.
3. The Fanning friction factor
 - has units of length.
 - applies only to irrotational flows.
 - is independent of Reynolds number.
 - is a non-dimensional version of head loss.
 - can be found by solving a differential equation.

2 (5 points) Describe the Reynolds tube experiment, explaining the different regimes and the role of the Reynolds number.

3 (10 points) Water at 49 °F flows through a straight section of a 6-in.-ID cast-iron pipe with an average velocity of 4 fps. The pipe is 1200 ft long, and there is an increase in elevation of 1 ft from the inlet of the pipe to its exit. Find the power required to produce this flow rate for the specified conditions. [Material properties: $\rho = 62.3 \text{ lb}_m \text{ ft}^{-3}$, $\nu = 1.22 \times 10^{-5} \text{ ft}^2 \text{ s}^{-1}$. Use the transition formula

$$\frac{1}{\sqrt{f_f}} = 4 \log_{10} \frac{D}{e} + 2.28 - 4 \log_{10} \left(4.67 \frac{D/e}{\text{Re} \sqrt{f_f}} + 1 \right).$$

Cast iron: $e = 0.00085 \text{ ft}$. Also $1 \text{ hp} = 550 \text{ lb}_f \text{ ft s}^{-1}$.]

4 (12 points) Carry out the momentum integral analysis for the case of no pressure gradient with $v_x = a \sin by$ (you will have to solve for a and b .) Compute the momentum and displacement thicknesses. [The governing equation is

$$\tau = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \rho U^2 \frac{d}{dx} \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy.$$

You may find the following integrals useful:

$$\int_0^d \sin \frac{\pi y}{2d} \left(1 - \sin \frac{\pi y}{2d}\right) dy = d \frac{4 - \pi}{2\pi},$$
$$\int_0^d \left(1 - \sin \frac{\pi y}{2d}\right) dy = d \frac{\pi - 2}{\pi}.$$