1) $Q = 5\ m^3/s$; $D = 0.2\ m$; Smooth Pipe
Glycerin (at 20°C): $\nu = 10^{-3}\ m^2/s$; $\rho = 1800\ kg/m^3$; $\mu = 1.8\ kg/m\cdot s$

$Q = \bar{u} \cdot A$; $\bar{u} = \frac{Q}{A} = 159.2\ m/s = \bar{V}_x$

$Re = 31840$, can use eq 13.26.

$$\tau_0 = 0.0225\ \rho \bar{V}_{x\,max}^2 \left(\frac{\nu}{\bar{V}_{x\,max} \cdot y_{max}} \right)^{1/4}$$

still need $\bar{V}_{x\,max}$; use power law eqn's for $n = 7$.

$$\frac{\bar{V}_x}{\bar{V}_{x\,max}} = \frac{2n^2}{(n+1)(2n+1)}$$ (see Fox & McDonald, p. 354)

so for $n = 7$: $\bar{V}_{x\,max} = 194.86\ m/s$.

$y_{max} = R = 0.1\ m$.

so, $\sqrt{\frac{\tau_0}{\rho}} = 8.5\ m/s$;

- Laminar Sublayer : $y^+ = \frac{\sqrt{\tau_0/\rho}}{\nu} \cdot y$

$y^+ = 5$; $\Rightarrow$ $\boxed{y = 0.588\ mm.}$

- Buffer Layer : $5 \le y^+ \le 30$; $y^+ = 30$
  $y = 3.528\ mm$; So $\Delta y = 3.528 - 0.588$; $\boxed{\Delta y = 2.94\ mm.}$

- Turbulent Core :
  $R - \Delta y = 100 - 2.94 = \boxed{97.06\ mm.}$

14.6)

- Δp main to pipe.

$$\frac{P_m}{\rho} = \frac{V_p^2}{2} + \frac{P_p}{\rho} ; \qquad \boxed{\frac{\Delta P_1}{\rho} = \frac{V_p^2}{2}}$$

- Δp from elbows and valves.

$$\frac{\Delta P_2}{\rho} = \underbrace{\Sigma K \frac{V_p^2}{2}}_{\text{due to fittings}} + \underbrace{2 f_f \frac{L}{D} V_p^2}_{\text{due to pipe length.}}$$ (see p 190 and 193)

$$\boxed{\frac{\Delta P_2}{\rho} = \frac{V_p^2}{2}\left[\Sigma K + 4 f_f \frac{L}{D}\right]}$$

- Δp due to nozzle.

$$\frac{V_p^2}{2} + \frac{P_p}{\rho} = \frac{V_N^2}{2} + \frac{P_{atm}}{\rho} ;$$

$$\frac{\Delta P_3}{\rho} = \frac{V_N^2}{2} - \frac{V_p^2}{2} = \frac{V_p^2}{2}\left(\frac{V_N^2}{V_p^2} - 1\right)$$

Apply conservation of mass: $A_p V_p = A_N V_N$;

$$\frac{V_N}{V_p} = \frac{A_p}{A_N}$$; use in eqn above.

$$\boxed{\frac{\Delta P_3}{\rho} = \frac{V_p^2}{2}\left(\frac{A_p^2}{A_N^2} - 1\right)} ;$$

- total pressure change : $\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3$

$$\boxed{\frac{\Delta P}{\rho} = \frac{V_p^2}{2}\left(\Sigma K + 4 f_f \frac{L}{D} + \left(\frac{A_p}{A_N}\right)^2\right)}$$

find ΣK: 90° elbow : $K = 0.7$
wide open angle valve: $K = 3.8$
globe valve: $K = 7.5$
} see table 14.1.

so, $\Sigma K = 6 \cdot 0.7 + 3.8 + 7.5 = 15.5.$

$$\left(\frac{A_p}{A_N}\right)^2 = 19.52;$$

$\Delta P = 60 \text{psig} = 161\ lb_m/in^2$
$\rho = 62.3\ lb_m/ft^3 = 0.0361\ lb_m/in^3$

Plug in all the known values into Δp eqn:

$$V_p = \sqrt{\frac{8919.7}{35.02 + 10240 f_f}};$$

use f_f for first estimate: $f_f = 0.007 \Rightarrow V_p = 9.14$ ft/s
$Re = 34621.2.$
for copper pipe (drawn tube): $e = 5 \cdot 10^{-6}$ ft (see fig 14.2),
so $\frac{e}{D} = \frac{5 \cdot 10^{-6}}{(0.75/12)} = 0.00008;$

use eq 14.15a: $$\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{e}{D} \cdot \frac{1}{3.7}\right)^{10/9}\right];$$

new $f_f = 0.00568;$

Repeat all computations with $f_f = 0.00568$: $V_p = 9.785$ ft/s
$Re = 37064.2$; and obtain new $f_f \Rightarrow \underbrace{f_f = 0.00559}_{\text{close enough to above}}$

So, $V_p = 9.785$ ft/s;
$Q = A_p \cdot V_p$; $\boxed{Q = 0.03\ ft^3/s}$

14.8) $0 = \dfrac{P_2 - P_1}{\rho} + h_L g$;

$\dfrac{\Delta P}{\rho} = \dfrac{kV^2}{2}$; $P_1 = 236$ kPa

$P_2 = 101$ kPa (P_{atm}) :

$Q = A \cdot V$; $V^2 = \dfrac{Q^2}{A^2} = \dfrac{Q^2}{(\pi D^2/4)^2} = \dfrac{16\,Q^2}{\pi^2 D^4}$

$\dfrac{\Delta P}{\rho} = \dfrac{8 k Q^2}{\pi^2 D^4}$;

$Q^2 = \dfrac{\pi^2 D^4 (P_1 - P_2)}{8 \rho K}$; $D = 0.2$ m.

Assume : water at $10°C$ => $\rho = 1000$ kg/m^3

Gate Valve	K	Q
Open	0.15	1.77
1/4 Closed	0.85	0.314
1/2 Closed	4.4	0.061
3/4 Closed	20	0.0133

14.15) $\Delta P = 210$ kPa; $l_{eq} = 150$ m;
Cast iron: $D = 0.2$ m; $e = 0.000259$ m
Commercial steel: $D = 0.067$ m; $e = 0.000046$ m

$$\frac{\Delta P}{\rho} = 2 f_f \frac{L_{eq}}{D} V^2; \quad \Delta p = 2\rho f_f \frac{L_{eq}}{D} V^2; \quad Re = \frac{VD}{\nu};$$

Plug in all the common values:

$$f_f = \frac{0.7D}{V^2}; \quad Re = \frac{V \cdot D}{1.3 \cdot 10^{-6}}$$

Cast iron: $f_f = 0.14/V^2$; $Re = 153846\,V$; $\frac{e}{D} = 0.001295$

use eq 14.15a to find f_f:

$$\frac{V}{\sqrt{0.14}} = -3.6 \log_{10}\left[\frac{6.9}{153846V} + 0.000145\right]$$

$$V = -1.347 \log_{10}\left[\underbrace{\frac{0.000045}{V}}_{\text{small compared to}} + 0.000145\right], \text{ so}$$

$V = -1.347 \log_{10}[0.000145]$; $V = 5.17$ m/s ; $\boxed{Q = 0.162 \frac{m^3}{s}}$

Commercial steel: $f_f = 0.0469/V^2$; $Re = 51538.5\,V$;
$e/D = 0.000687$

use 14.15a again: $$\frac{V}{\sqrt{0.0469}} = -3.6 \log_{10}\left[\frac{6.9}{51538.5V} + 0.000071\right];$$

$$V = -0.7796 \log_{10}\left[\frac{0.000134}{V} + 0.000071\right];$$

Assume again $\frac{0.000134}{V} \ll 0.000071$ (bad assumption)

$V = 3.23 \text{ m/s} \Rightarrow Re = 166701.$

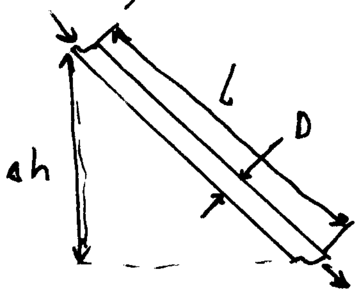
~~we~~ run the itteration again : $\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[0.000113\right]$;

$f_f = 0.004952.$

Use in $V^2 = \frac{0.0469}{f_f}$ to obtain $V = 3.08 \text{ m/s}$

$Q = \frac{\pi D^2}{4} \cdot V$; $\boxed{Q = 0.0109 \frac{m^3}{s}}$

14.18)



$\Delta h = 668\ m$
$D = 5\ m$
$L = 8000\ m$
$Q = 90\ m^3/s$
$e = 0.01\ ft$ (concrete pipe)

$$\Delta P = P_2 - P_1 = \rho g \left(-h_L + \Delta h\right)$$

$$\Delta P = \rho g \Delta h - 2\rho f_f \frac{L_{eq}}{D} V^2 ;$$

$$V = \frac{Q}{A} = 4.584\ \frac{m}{s};$$ use all know values above.

$$\Delta P = 6.553 \cdot 10^6 - 6.723 \cdot 10^7 f_f ;$$

$Re = 1.763 \cdot 10^7$ (assume water at $10°C$)

$\frac{e}{D} = 0.00061$; (convert to meters).

use eq 14.15a:

$$\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[\frac{6.9}{1.763 \cdot 10^7} + \left(\frac{0.00061}{3.7}\right)^{10/9}\right];$$

$f_f = 0.0044$: use in ΔP eqn:

$$\boxed{\begin{aligned} \Delta P &= 6.26 \cdot 10^6\ Pa \\ \Delta P &= 6.26 \cdot 10^3\ kPa \end{aligned}}$$