CENG101A: Introductory Fluid Mechanics Fall Quarter 2006
http://maecourses.ucsd.edu/mae210a

Solutions III.

1 Consider a small cylindrical volume of fluid. Then from mass conservation $\rho A v$ is constant along the volume. Take the differential

$$Av\, dd\rho + (\rho v)\,\mathrm{d}A + (\rho A)\,\mathrm{d}v = 0$$

and divide by $\rho A v$. Newton II gives $F_1 - F_2 = -\rho A_1 v_1^2 + \rho A_2 v_2^2$, i.e. the change in F is given by

$$\mathrm{d}F = -\mathrm{d}(\rho A v^2) = -\mathrm{d}(\dot{m} v) = -\dot{m}\,\mathrm{d}v.$$

This is the force exerted on the fluid. The force exerted by the fluid is the opposite.

2 Mass conservation in the rectangular control volume gives

$$-\rho w U \delta + \dot{m} + \rho w \int_0^\delta u(y)\,\mathrm{d}y = 0,$$

where $\dot{m}$ is the mass flux out of the boundary layer. This gives

$$\dot{m} = \rho w U \delta - \rho w U \int_0^\delta (2y/\delta - y^2/\delta^2)\,\mathrm{d}y = \rho w U \delta/3.$$

So fluid is leaving the boundary layer. In numbers $\dot{m} = 1.24 \times 0.6 \times 30 \times 5 \times 10^{-3}/3 = 0.0372$ kg s^{-1}.

3 Newton II in the same control volume in the x-direction (along the boundary) gives (assuming no pressure gradient across the volume)

$$-\rho w U^2 \delta + U\dot{m} + \rho w \int_0^\delta u(y)^2\,\mathrm{d}y = F_x.$$

The tricky bit here is the second term: the momentum flux across the top of the boundary layer where the velocity is U. The force on the boundary is $-F_x$, i.e.

$$(2/3)\rho w U^2 \delta - \rho w U^2 \int_0^\delta (2y/\delta - y^2/\delta^2)^2\,\mathrm{d}y = (2/15)\rho w U^2 \delta.$$

This force is positive: there is a force in the direction of the flow on the plate. Its value is 0.4464 N. Note that is the actual force, not the force per unit area.

4 Steady frictionless flow. Force parallel to plate is from Newton II

$$F = U \cos\theta \, \dot{m},$$

since the flow is oriented at an angle θ to the plate. The horizontal components and vertical components of the flow are hence

$$F_x = U \cos^2\theta \, \dot{m}, \qquad F_y = U \sin\theta \cos\theta \, \dot{m}.$$

So $F_x = 2F_y$ implies $\cos\theta = 2\sin\theta$, i.e. $\tan\theta = 1/2$, which gives $\theta = 26.6°$.

5 Consider a control volume bounded by the radii r_1 and r_2 with depths and velocities h_1, h_2 and v_1, v_2 respectively. Conservation of mass gives

$$2\pi r_1 \rho h_1 v_1 = 2\pi r_2 \rho h_2 v_2.$$

For momentum, we need to take a wedge-shaped volume with angle $\Delta\theta$, otherwise the forces integrate to zero by symmetry. Neglect frictional effects so the only forces acting are pressure forces. Newton II in the radial direction then gives

$$\Delta\theta(r_1 \tfrac{1}{2}\rho g h_1^2 - r_2 \tfrac{1}{2}\rho g h_2^2) = \Delta\theta(-r_1 \rho h_1 v_1^2 + r_2 \rho h_2 v_2^2).$$

The $\Delta\theta$ terms cancel, but the r_i terms survive. We obtain the following two equations

$$r_1 h_1 v_1 = r_2 h_2 v_2, \qquad \tfrac{1}{2} g r_1 h_1^2 - \tfrac{1}{2} g r_2 h_2^2 = -r_1 h_1 v_1^2 + r_2 h_2 v_2^2.$$

These two equations are unpleasant to solve. In the two-dimensional case, the r_i terms go away and progress can be made.