Solutions IV.

1 Frictionless flow, no change in temperature, horizontal pipe, constant diameter pipe, constant $\mathrm{d}P/\mathrm{d}z$. Bernoulli gives $h_L = -(L/\rho)\mathrm{d}P/\mathrm{d}z$: heat is removed from the water. If the pipe is insulated, this heat is not removed but acts to change the temperature. Use $u = cT$ and obtain $\Delta T = (L/c\rho)\mathrm{d}P/\mathrm{d}z$.

2 Assume no change in internal energy, no heat added to the system and no change in kinetic energy (we are not told anything about velocity). Then the shaft work is due to the change in potential energy and we have $h = \dot{W}_s/\dot{m}$, so

$$\dot{m} = \frac{\dot{W}_s}{h} = \frac{\dot{W}_s}{\eta h} = \frac{2.8 \times 10^9}{0.8 \times 726 \times 0.3} = 1.6 \times 10^7 \mathrm{m}^3\ \mathrm{s}^{-1}.$$

3 Assume frictionless. Apply Bernoulli from base of the fountain 1 to the top where $V_2 = 0$; note that we take the pressure to be atmospheric at the base and at the top. Then $V_1 = \sqrt{2gh} = 124.3$ ft s^{-1}.

4 The upper surface is at z and has velocity v. The exit is at 0 and has velocity v_0. By mass conservation we have

$$\pi h(z)^2 v = \pi h(0)^2 v_0.$$

Apply steady Bernoulli's equation (this assumes that the changes in height are small in some sense). Assume that at the outlet the flow comes out as a constant-diameter jet so that the pressure there is atmospheric. Then we have

$$gz + \tfrac{1}{2}v^2 = \tfrac{1}{2}v_0^2.$$

Eliminate v_0 from these two equations:

$$gz + \tfrac{1}{2}v^2 = \tfrac{1}{2}v^2 \frac{h(z)^4}{h(0)^4}.$$

We require v to be constant in time, so this is not an ordinary differential equation but just an algebraic relation. The profile $h(z)$ is given by

$$h(z) = h(0)\left(1 + \frac{2gz}{v^2}\right)^{1/4}.$$

5 At the bottom of the gap the fluid is at rest, at the top it has velocity U so the velocity profile in the gap is $u = Uy/d$. The shear stress is $\mu \mathrm{d}u/\mathrm{d}y$, which is $\mu U/d$ here. Force is stress times area so $F = \mu UA/d$.