Solutions V.

1 A fluid that behaves as a solid until a minimum yield stress, τ_y, is exceeded and subsequently exhibits a linear relation between stress and rate of deformation is referred to as an ideal or Bingham plastic. The shear stress model is

$$\tau_{yx} = \tau_y + \mu \frac{\mathrm{d}u}{\mathrm{d}y}.$$

Clay suspensions, drilling muds, and toothpaste are examples. (From "Introduction to Fluid Dynamics", Fox & McDonald, 5th ed., Wiley).

2 The change in heat from class is

$$\frac{\delta Q}{\mathrm{d}t} = \mu \frac{r^2\omega^2(2\pi r l)}{\epsilon},$$

which scales like ω^2, so having the speed will divide the heat transfer by 4. This is a 75% decrease.

3 Note that $\nabla\cdot\mathbf{u} = 0$. The stress is

$$\begin{aligned}
\tau_{xy} &= -2\mu, \qquad \tau yz = 2\mu, \qquad \tau zx = 2\mu A,\\
\sigma_{xx} &= 6\mu - (-3z^2 + 2y + 1),\\
\sigma_{yy} &= -(-3z^2 + 2y + 1),\\
\sigma_{zz} &= -6\mu - (-3z^2 + 2y + 1).
\end{aligned}$$

The bulk stress is $-p$, which is independent of A.

3 Fully-developed flow in a pipe so

$$r\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}r}(r\tau_{rx}).$$

Integrate to give $\tau_{rx} = (r/2)\mathrm{d}P/\mathrm{d}x$, since we can't have a singularity at the axis. Now substitute constitutive relation and take cubic root to obtain

$$\frac{\mathrm{d}v_x}{\mathrm{d}r} = \left(\frac{r}{2\mu}\frac{\mathrm{d}P}{\mathrm{d}x}\right)^{1/3}.$$

This can be integrated directly to give

$$v_x = \left(\frac{1}{2\mu} \frac{\mathrm{d}P}{\mathrm{d}x} \right)^{1/3} \frac{3r^{4/3}}{4} + C.$$

The velocity vanishes at the walls so we have

$$v_x = \left(\frac{1}{2\mu} \frac{\mathrm{d}P}{\mathrm{d}x} \right)^{1/3} \frac{3(r^{4/3} - a^{4/3})}{4}.$$

The average velocity is

$$\bar{v} = \frac{2}{a^2} \int_0^a v_x r \, \mathrm{d}r = -\frac{3a^{4/3}}{10} \left(\frac{1}{2\mu} \frac{\mathrm{d}P}{\mathrm{d}x} \right)^{1/3}.$$

5 As in class, we have

$$u = \frac{y^2}{2\mu} \frac{\mathrm{d}P}{\mathrm{d}x} + \left(Uh - \frac{h^2}{2\mu} \frac{\mathrm{d}P}{\mathrm{d}x} \right) y.$$

The average velocity is proportional to the volume flux

$$Q = \int_0^y u(y) \, \mathrm{d}y = U \frac{h^2}{2} - \frac{h^2}{12\mu} \frac{\mathrm{d}P}{\mathrm{d}x}.$$

For this to vanish, we need $U = (h^2/6\mu)\mathrm{d}P/\mathrm{d}x$.