Solutions VII.

1 The streamlines are hyperbolae. The vorticity is $\omega = -\nabla^2\psi = 4$. This does not vanish so the flow is not irrotational and there is no velocity potential.

2 The boundary of the ellipse is a streamline with $\psi = 1$. Flow is tangent to a streamline, so the boundary has no normal flow through it. This is the appropriate boundary condition for inviscid fluid. The vorticity of the flow is $\omega = -\nabla^2\psi = -2/a^2 - 2/b^2$.

3 In spherical polar coordinates this is

$$\phi = U\cos\theta\left[r + \frac{a^3}{2r^2}\right].$$

The radial velocity component is

$$u_r = \frac{\partial\phi}{\partial r} = U\cos\theta\left[1 - \frac{a^3}{r^3}\right],$$

which vanishes on $r = a$. Hence there is no normal flow through the boundary. At infinity, $\phi \sim Ux$, which corresponds to uniform flow $(U, 0)$. Hence this is the potential for flow past a sphere.

4 As in class, we use the tangential velocity component on the boundary of the cylinder, $u_\theta = 2U_\infty\sin\theta$. The pressure field from Bernoulli is $p = p_\infty - 2\rho U_\infty^2\sin^2\theta$. The drag is the force parallel to the flow, so we need to use $n_x = \cos\theta$. Then we have

$$D = -\int pn_x\,\mathrm{d}S = -\int_0^{2\pi}[p_\infty - 2\rho U_\infty^2\sin^2\theta\cos\theta]a\,\mathrm{d}\theta = 0,$$

an example of d'Alembert's paradox. Along the front half of the cylinder, we have

$$D == -\int_{-\pi/2}^{pi/2}[p_\infty - 2\rho U_\infty^2\sin^2\theta\cos\theta]a\,\mathrm{d}\theta = -2ap_\infty + \frac{4}{3}a\rho U_\infty^2 a.$$

(These are all forces per unit length.)

5 Water at 10 C: $\rho = 1,000$ kg m^{-3}, $\mu = 1.3 \times 10^{-3}$ N s m^{-2}, $c = 1450$ m s^{-1} (speed of sound). Also $g = 9.81$ m s^{-2}. Take $L = 317$ m (waterline of USS Nimitz), $U = 15.6$ m s^{-1}. Then the three nondimensional numbers are

$$\begin{aligned}
\mathrm{Re} &= \frac{\rho U L}{\mu} = 3.8 \times 10^{9}, \\
\mathrm{Fr} &= \frac{U}{\sqrt{gL}} = 0.28, \\
\mathrm{Ma} &= \frac{U}{c} = 0.011.
\end{aligned}$$