Solutions VIII.

1 Five parameters: Δp [ML^{-1}T^{-2}], D [L], ρ [ML^{-3}], ω [T^{-1}], Q [MT^{-1}]. Three dimensions. Two non-dimensional parameters. No unique choice; take ρ, D and ω as repeating parameters. Get

$$\pi_1 = \frac{\Delta p}{\rho D^2 \omega^2}, \qquad \pi_2 = \frac{Q}{\rho D^3 \omega}.$$

2 Six parameters: t [T], ℓ [L], d [L], D [L], μ [ML^{-1}T^{-1}], $\Delta\gamma$ [ML^{-2}T^{-2}]. Three dimensions. Three non-dimensional parameters. No unique choice, although there are two obvious geometric ratios. Take e.g.

$$\pi_1 = \frac{Dt\gamma}{\mu}, \qquad \pi_2 = \frac{\ell}{D}, \qquad \pi_3 = \frac{d}{D}.$$

Since $\pi_1 = f(\pi_2, \pi_3)$, obtain

$$t = \frac{\mu}{\gamma} D\gamma f\left(\frac{\ell}{D}, \frac{d}{D}\right).$$

Hence if one takes a fluid of known viscosity and specific weight and carries out the experiment, one obtains a number π_1 that depends on the dimensions of the apparatus. Now if one takes a fluid with unknown viscosity but known specific density and carries out the experiment, obtain t. Then using the known value of π_1 can obtain μ.

3 Take $\mathrm{Fr} = U/\sqrt{gH}$ (this is not the only possible definition). The value of g is fixed, while $H_m = 0.1$ m and $Hp = 3$ m. We require $\mathrm{Fr}_m = \mathrm{Fr}_p$, so $U_p = U_m\sqrt{H_p/H_m} = 2\sqrt{30} = 10.95$ m s^{-1}.

4 Take transition on flat plate to occur at $\mathrm{Re} = 5 \times 10^5$. Here $U = 20$ m s-1, $\rho = 1000$ kg m^{-3} and $\nu = 1.3 \times 10^{-6}$ m^{2}s^{-1} (at 10 °C). So

$$L = \frac{\mathrm{Re}\nu}{U} = \frac{1.3 \times 10^{-6} \times 5 \times 10^5}{20} = 0.0325\text{m}.$$

5 Assume effects of ground are negligible and that cross-section is cylindrical. Then the drag coefficient is $C_D = 0.35$ (approximately – see Fig. 12.2 on p. 151). Use $D = 6$ ft, $U = 20$ mph $= 29.3$ ft s^{-1}, $H = 20$ ft, $\nu = 1.74 \times 10^{-4}$ ft^2 s^{-1} and $\rho = 0.00225$ slugs ft^{-3} (at 90 °F). Now $F_D = \frac{1}{2}\rho U^2 A C_D$, where $A = HD$ is the projected area. Hence $F_D = 40.56$ lb$_f$.