CENG101B Heat Transfer Fall Quarter 2008

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Quiz II

This is a 50 minute closed-book exam; no notes. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. The heat equation
   - is the same as the Navier–Stokes equation.
   - cannot be solved.
   - includes h.
   - can be derived from the First Law of Thermodynamics.
   - only holds in a vacuum.
2. For one-dimensional steady-state conduction,
   - the temperature gradient is constant.
   - the temperature is zero.
   - the solution is independent of the boundary conditions.
   - heat generation is always important.
   - the heat flux decays like $1/r$.
3. Fourier's equation
   - is a conservation law.
   - is a constitutive law that relates heat flux to temperature gradient.
   - governs radiative heat transfer.
   - is a reformulation of the First Law of Thermodynamics.
   - always leads to steady-state problems.

2 (7 points) The material "Quadboard" has a thermal conductivity that is not constant, but that behaves as $k = \alpha(T - T_0)^2$. What are the units of α? For a steady one-dimensional problem, what is the heat flux? Write down the governing equation for temperature. Solve it for the case where $T = T_0$ at $x = 0$ and $T = 2T_0$ at $x = L$. Evaluate the heat flux q. What constant value of k gives the same value of q? [Note: the ODE $\theta^2 d\theta/dx = C$ can be solved by separating variables.]

$$\underset{\frac{W}{mK}}{k} \overset{?}{=} \alpha \underset{K^2}{(T - T_0)^2} \Rightarrow \alpha \text{ is } \frac{W}{mK^3}$$

Heat flux: $q_x = -k\dfrac{dT}{dx} = -\alpha(T - T_0)^2 \dfrac{dT}{dx}$

steady problem, no heat generation: $q_x =$ constant $= C$

$$\Rightarrow -\alpha(T - T_0)^2 \frac{dT}{dx} = C$$

Separate variables: $-\alpha(T - T_0)^2 dT = C dx$

$$\Rightarrow -\frac{1}{3}\alpha(T - T_0)^3 = Cx + D$$

BCs: $\begin{cases} 0 = D \\ -\frac{1}{3}\alpha(2T_0 - T_0)^3 = CL + D \end{cases}$ i.e. $-\frac{1}{3}\alpha T_0^3 = CL$

So $-\frac{1}{3}\alpha(T - T_0)^3 = -\frac{1}{3}\alpha T_0^3 \dfrac{x}{L}$

Flux $q_x = C = -\dfrac{\alpha T_0^3}{3L}$

Constant k: $T = Ax + B$ — BCs: $\begin{array}{l} T_0 = B \\ 2T_0 = AL + B \\ \hline T_0 = AL \end{array}$

so $q_x = -k_e A = -k_e \dfrac{T_0}{L}$

Equate: $-k_e \dfrac{T_0}{L} = -\dfrac{\alpha T_0^3}{3L} \Rightarrow k_e = +\dfrac{\alpha T_0^2}{3}$

3 (10 points) Hot water at 70°C flows along a 2-cm diameter copper pipe. The outside air temperature is 25°C. If the pipe is wrapped in 2 cm of fiberglass ($k = 0.036$ Wm^{-1}K^{-1}), what is the heat flow per unit meter of pipe? How much insulation is required to reduce the heat flow by a factor of two? If there is a 1-mm thick layer of scale (essentially calcium carbonate with $k = 2.5$ Wm^{-1}K^{-1}) on the inside of the pipe, what does the heat flux become? Justify in words why you have ignored the effect of the copper ($k = 400$ Wm^{-1}K^{-1}) pipe.

Cylinder: $R = \dfrac{\ln r_o/r_i}{2\pi k L}$

$$Q = \frac{\Delta T}{\sum R}$$

(a) Fiberglass from 1 to 3 cm radius

$$\frac{Q}{L} = \frac{2\pi k \Delta T}{\ln r_o/r_i} = \frac{2\pi \times 0.036 \times 45}{\ln 3} = 9.2651 \ \frac{W}{m}$$

(b) Divide by 2? $\ln \dfrac{r_{new}}{r_i} = 2 \ln \dfrac{r_{old}}{r_i} \rightarrow \dfrac{r_{new}}{r_i} = \left(\dfrac{r_{old}}{r_i}\right)^2 = 9$

i.e. 8 cm thickness

(c) Calcium carbonate from 0.9 to 1 cm

$$\frac{Q}{L} = \frac{2\pi \Delta T}{\dfrac{\ln r_o/r_i}{k_f} + \dfrac{\ln r_i/r_s}{k_s}} = \frac{2\pi \times 45}{\dfrac{\ln 3}{0.036} + \dfrac{\ln \frac{1}{0.9}}{2.5}} = 9.2523 \ \frac{W}{m}$$

(d) copper pipe: $k = 400 \ \dfrac{W}{m K}$

so $\dfrac{\ln r_2/r_1}{k_c} \ll 1$ unless $\dfrac{r_2}{r_1}$ big

(not true for pipe)