CENG101B Heat Transfer

Fall Quarter 2008
http://maecourses.ucsd.edu/ceng101b

Quiz II

This is a 50 minute closed-book exam; no notes. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. The effectiveness of a finned heat exchanger with no-flux end condition
   - is independent of its width.
   - tends to 0 as the fin becomes longer.
   - can be computed in two ways: by integrating over the base or by integrating over the surface.
   - is the same for all fins of the same shape.
   - was derived in class for short fins.
2. For small Biot numbers,
   - the temperature along the boundary of the solid is equal to the ambient temperature.
   - temperature changes linearly in time inside the solid.
   - the solution one uses is only valid for small X_{Fo}.
   - one can find the solution by solving an ordinary differential equation.
   - the solution is independent of the ambient temperature.
3. The unsteady heat equation has a similarity solution for a semi-infinite solid
   - because there is no natural length scale for the problem.
   - because there is no time dependence
   - for narrow solids.
   - that holds in a narrow layer near the boundary.
   - that is independent of the thermal diffusivity α.

2 (7 points) A 52&7/8-inch long, 2-inch wide and 1/2-inch thick steel blade is put in a fire to obtain an impressive effect for a blockbuster Hollywood trilogy. Given that a bright cherry-red glow is visible at 1000°C and that the fire is at 1600° C, how long does it take for the sword to reach a photogenic temperature? [The properties of steel vary over such a large temperature range; take the values (actually for iron) at 500°C of $k = 61$ W/m·K, $c_p = 532$ J/kg·K and $\rho = 7,870$ kg/m^3 as representative. Assume there is little air flow directly past the blade, so that the heat transfer coefficient has the value of 20 W/m^2·K.]

Compute Bi using $R = \frac{1}{2} \times \frac{1}{2}$ in (smallest dimension counts)

Then $Bi = \dfrac{20 \times \frac{1}{4} \times 0.0254}{61} = 0.0021 \ll 1$

Use $Bi \ll 1$ solution $\theta = \exp\left(-\dfrac{hAt}{\rho C_p V}\right) = e^{-t/\tau}$

Here $V = 52\,^7/_8 \times 2 \times \frac{1}{2}$ in^3 $= 52.875$ in^3

$A = 2 \times \left(52\,^7/_8 \times 2 + 52\,^7/_8 \times \frac{1}{2} + 2 \times \frac{1}{2}\right)$ in^2 $= 266.375$ in^2

So $\tau = \dfrac{\rho C_p V}{hA} = \dfrac{7870 \times 532 \times 52.875}{20 \times 266.375} \times \dfrac{0.0254\,\text{m}}{1\,\text{in}^3}$

$\simeq 1056$ s

$\theta = \dfrac{T - T_a}{T_0 - T_a} = \dfrac{1000 - 1600}{25 - 1600} = 0.38$

so $t = -\tau \ln \theta = 1020$ s $\simeq 17$ minutes

3 (10 points) A thin rectangular slab of silicon of half-thickness 10-cm (thermal diffusivity $\alpha = 0.8\ \text{cm}^2/\text{s}$) at 80°C is immersed in a bath of water that is maintained at 4°C. What are the temperatures at the midpoint and surface of the slab after 1 second? After 1 minute? [The similarity solution for transient heat conduction with an imposed temperature at $y = 0$ in the semi-infinite slab $y > 0$ is given by $\Theta = \text{erf}(y/\sqrt{4\alpha t})$. The thermal penetration depth is $\delta_T = 4(\alpha t)^{1/2}$. The temperature in a thin slab for not-too-short times is given by $\Theta_1 = A_1 \exp(-\lambda_1^2 X_{Fo}) \cos\lambda_1\xi$, where ξ is the non-dimensional coordinate measured from the midpoint of the slab ($\xi = 0$) to its surface ($\xi = 1$). See the graphs on the next page.]

Water temperature constant $\Rightarrow T_s = 4°C$ always

$Bi \gg 1$! (see graphs!)

$$X_{Fo} = \frac{\alpha t}{R^2} = \frac{0.8\ \text{cm}^2/\text{s} \times \begin{Bmatrix} 1 \\ 60 \end{Bmatrix} \text{s}}{(10\ \text{cm})^2} = \begin{cases} 0.008 \ll 1 \\ 0.48 = O(1) \end{cases}$$

A) $t = 1$ – $X_{Fo} \ll 1$ – use erf sol^n.

Thermal penetration depth $\delta_T = 4\left(0.8\ \text{cm}^2/\text{s} \times 1\text{s}\right)^{\frac{1}{2}} = 3.6\ \text{cm}$

so midplane hasn't felt flux yet and $T_c = 80°C$

$T_s = 4°C$

B) $t = 60\text{s}$ – $X_{Fo} = \cancel{0.48}$ 0.48. Use Fig 11.1.4 (b) : $\Theta_1^0 \simeq \cancel{0.14}$ 0.25

$T_c : \quad T_a + \Theta_1^0 (T_0 - T_a) = 4 + \cancel{0.14}\ 0.25\,(80 - 4) = \cancel{14.64°C}$ 23

$T_s = 4°C$