CENG101B Heat Transfer Fall Quarter 2008
http://maecourses.ucsd.edu/ceng101b

Quiz III

This is a 50 minute closed-book exam; no notes. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. The Nusselt number
   - is independent of Reynolds number.
   - depends only on the material.
   - applies only to laminar flows.
   - is a non-dimensional version of the heat transfer coefficient.
   - is always less than 1.
2. Analogies for heat transfer
   - relate the Nusselt or Stanton number to the other non-dimensional parameters of the flow.
   - can always be obtained from boundary-layer analysis.
   - are independent of the geometry of the system.
   - are not useful for solving unsteady heat conduction problems.
   - are independent of Prandtl number.
3. One-term approximations for laminar heat transfer problems
   - are valid for $Re > 3000$.
   - are independent of geometry.
   - are valid sufficiently far from the entrance.
   - are obtained empirically.
   - hold near the entrance.

2 (7 points) A viscous oil (Pr = 180, $k = 0.2$ W/m·K, $\mu = 1.2$ Pa·s, $\rho = 920$ kg/m^3) is pumped through a circular pipe of diameter $D = 1.5$ cm, the surface of which is maintained at 500 K. What length of pipe is required to permit a flow of 8 m^3/h to be raised from 350 to 400 K?

$$Re_D = \frac{4w}{\pi \cdot D \cdot \mu} = \frac{4 \cdot (8/3600) \cdot 920}{\pi \cdot 0.015 \cdot 1.2} = 144.6 \Rightarrow \text{laminar.} \quad Pr = 180$$

$$\theta_{cm} = \frac{T_2 - T_s}{T_1 - T_s} = \frac{400-500}{350-500} \simeq 0.67 = 0.82\exp(-3.66 z^*) \Rightarrow z^* = 0.055$$

$$z^* = \frac{\alpha z}{UR^2} = \frac{4(z/D)}{Pr \cdot Re_D}$$ cannot be used because $z^* = 0.055 \ll 0.3$

use dimensionalization.

$$q_w = h(T_w - T_{cm}) = k\frac{\partial T(R,z)}{\partial r} \Rightarrow \frac{hR}{k}(\theta_w - \theta_{cm}) = \frac{\partial \theta(1, z^*)}{\partial r^*} \qquad \left(r^* = \frac{r}{R}\right)$$

$$Nu = \frac{hD}{k} = \frac{2}{\theta_w - \theta_{cm}}\frac{\partial \theta(1, z^*)}{\partial r^*} \quad \text{where } \theta_w(z^*) = \theta(1, z^*).$$

$$U\rho c_p \pi R^2\, dT_{cm} = h\, 2\pi R(T_w - T_{cm})\, dz \qquad T_{cm}(z=0) = T_0.$$

$$\Rightarrow d\theta_m = Nu(\theta_w - \theta_{cm})\, dz^* \qquad \theta_{cm}(z^*=0) = \theta_{cm_0}$$

$$\Rightarrow \frac{d\theta_{cm}}{\theta_{cm}} = Nu\, dz^* \qquad \theta_{cm}(z^*=0) = 1 \quad (\text{because } T_r = T_w \ \&\ \Delta T = T_0 - T_w \Rightarrow \theta_w = 0)$$

$$\theta_{cm} = \frac{T_{cm} - T_w}{T_0 - T_w} = \exp\left[-\int_0^{z^*} Nu\, dz^*\right]$$

$$\overline{Nu} = \frac{1}{z^*}\int_0^{z^*} Nu\, dz^*$$

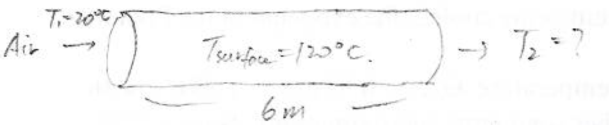
$$\overline{Nu} = -\frac{\ln(\theta_{cm})}{z^*} = 1.614\left(\frac{L/D}{Pr \cdot Re_D}\right)^{-1/3}$$

$$-\frac{\ln(0.67)}{0.055} = 1.614\left(\frac{L/0.015}{180 \cdot 144.6}\right)^{-1/3} \Rightarrow L = 4.25\ (m)$$

3 (10 points) Explain the assumptions that lead to the result for cup-mixing temperature in pipe flow:

$$\frac{T - T_R}{T_1 - T_R} = \exp\left(-\frac{\pi D h z}{w C_p}\right).$$

Define the cup-mixing temperature.
Air at 20°C flows through a 6 m length of tube of diameter 5 cm at a mass flowrate of 30 kg/h. The tube is held at 120°C. Find the temperature at the end of the pipe, using the Colburn analogy and the Dittus–Boelter equation.



$D = 0.05\,(m)$ $\quad w = 30/3600\ (kg/s)$

① assumptions.
- flow field is 1-D. ($u = [u_z(r), 0, 0]$).
- the system has axial symmetry
- steady state conditions
- physical properties are independent of temperature

② cup-mixing temperature : the temperature of the fluid that we would measure if we collected the efflux from the pipe in a cup and mixed the contents to yield a single uniform temperature

③ Assume $T_{out} = \left(\frac{T_1 + T_{surface}}{2}\right) = 70°C$.

colburn.

$\mu = 0.4 \times 10^{-3}\ Pa\cdot s \qquad Pr = 2.5$

$C_p = 4\ kJ/kg\cdot K = 4000\ J/kg\cdot K$

$k = 0.7\ W/m\cdot K$.

$Re = \frac{4w}{\pi D \mu} = \frac{4 \times \frac{30}{3600}}{\pi \cdot 0.05 \cdot 0.4 \times 10^{-3}} = 530 \Rightarrow f = 0.079\, Re^{-0.25} = 0.0165$

$Nu = \frac{f}{2} \cdot Re \cdot Pr^{\frac{1}{3}} = \frac{1}{2}(0.0165)(530)(2.5)^{1/3} = 5.93 \Rightarrow h = \frac{k}{D} \cdot Nu = 83.02$

$\frac{T_2 - 120}{20 - 120} = \exp\left(-\frac{\pi \cdot D \cdot h \cdot z}{w \cdot C_p}\right) \Rightarrow T_2 = 120 + (20 - 120)\exp\left(-\frac{\pi \cdot 0.05 \cdot (83.02) \cdot 6}{30/3600 \cdot 4000}\right)$

$= 110\ °C$.

④ D-B eqn

$St = 0.023\, Re^{-0.2}\, Pr^{-0.6} = 0.0038$.

$\frac{T_2 - T_s}{T_1 - T_s} = \exp\left(-\frac{4L}{D} \cdot St\right) \Rightarrow T_2 = 120 + (20 - 120)\exp\left(-\frac{4(6)}{0.05} \cdot 0.0038\right) = 104\ °C$