1.(1) parameters: $U_R$ [m/s], $k_p$ [W/m·K], Q [m³/s], H [m], $T_R$ [K], $T_A$ [K]

| | $U_R$ | $k_p$ | Q | H | $T_R$ | $T_A$ |
|---|---|---|---|---|---|---|
| M | 0 | 1 | 0 | 0 | 0 | 0 |
| L | 1 | 1 | 3 | 1 | 0 | 0 |
| T | -1 | -3 | -1 | 0 | 0 | 0 |
| K | 0 | -1 | 0 | 0 | 1 | 1 |

6 (all parameters) − 4 (rank) = 2 (nondimensional parameters)

Choose $k_p$, Q, H, $T_A$ as "repeating variables"

① $M^0L^0T^0K^0 = (MLT^{-3}K^{-1})^a (L^3T^{-1})^b (L)^c (K)^d (LT^{-1})$

M: $a=0$
K: $d=0$
T: $-b-1=0 \Rightarrow b=-1$
L: $3b+c+1=0 \Rightarrow c=2$

$\Pi_1 = \frac{U_R H^2}{Q}$ (4)

② $M^0L^0T^0K^0 = (MLT^{-3}K^{-1})^a (L^3T^{-1})^b (L)^c (K)^d (K)$

M: $a=0$
K: $d+1=0 \Rightarrow d=-1$
T: $-b=0 \Rightarrow b=0$
L: $3\cancel{b}+c=0 \Rightarrow c=0$

$\Pi_2 = \frac{T_R}{T_A}$ (1)

(2) parameters: Re, $\mu_w$ [N·s/m²], $\sigma$ [N/m], $D_p$ [m], $k_w$ [W/m·K], $k_o$ [W/m·K], $\mu_o$ [N·s/m²]

| | $k_w$ | $k_o$ | $\mu_w$ | $\mu_o$ | $\sigma$ | $D_p$ | Re |
|---|---|---|---|---|---|---|---|
| M | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| L | 1 | 1 | -1 | -1 | 0 | 1 | 0 |
| T | -3 | -3 | -1 | -1 | -2 | 0 | 0 |
| K | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

7 (all parameters) − 4 (rank) = 3 (nondimensional parameters)

① $\Pi_3 = Re$ (1)

Choose $k_w$, $\mu_w$, $\sigma$, $D_p$ as "repeating variables"

② $M^0L^0T^0K^0 = (MLT^{-3}K)^a (ML^{-1}T^{-1})^b (MT^{-2})^c (L)^d (MLT^{-3}K)$

K: $a+1=0 \Rightarrow a=-1$
M: $a+b+c+1=0 \Rightarrow b=-c$
T: $-3a-b-2c-3=0 \Rightarrow b=c=0$
L: $a-b+d+1=0 \Rightarrow b=d=0$

$\Pi_4 = \frac{k_o}{k_w}$ (1)

③ $M^0L^0T^0K^0 = (MLT^{-3}K)^a (ML^{-1}T^{-1})^b (MT^{-2})^c (L)^d (ML^{-1}T^{-1})$

K: $a=0$ M: $b+c+1=0$, T: $-b-2c-1=0$ $\Rightarrow c=0, b=-1$ L: $-b+d-1=0 \Rightarrow d=0$

$\Pi_5 = \dfrac{\mu_0}{\mu_w}$

(3) parameters: $Q[m^3/s]$, $D[m]$, $L[m]$, $T_a[K]$, $q[W/m^2]$, $a$

| | Q | D | L | $T_a$ | q | a |
|---|---|---|---|---|---|---|
| M | 0 | 0 | 0 | 0 | 1 | 0 |
| L | 3 | 1 | 1 | 0 | 0 | 0 |
| T | -1 | 0 | 0 | 0 | -3 | 0 |
| K | 0 | 0 | 0 | 1 | 0 | 0 |

6 (all parameters) − 4 (rank) = 2 (nondimensional parameters)

① $\Pi_6 = a$

Choose Q, D, $T_a$, q as "repeating variables"

② $M^0L^0T^0K^0 = (L^3T^{-1})^a (L)^b (K)^c (MT^{-3})^d L$

K: $c=0$ M: $d=0$ T: $a=0$ L: $b+1=0 \Rightarrow b=-1$

$\Pi_7 = \dfrac{L}{D}$

2. The differences between thermodynamics and heat transfer

The science of thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another equilibrium state, and no reference to how long the process will take. But in engineering, we are often interested in the rate of heat transfer, which is the topic of the science of heat transfer.

10.1


$h = 10\ (W/m^2 \cdot K)$

$k = 0.75\ (W/m \cdot K)$

$L = 0.5 \times 10^{-2}\ (m)$

$A = 1\ (m^2)$

$T_i = 25\,^\circ C ≒ 298\ (K)$

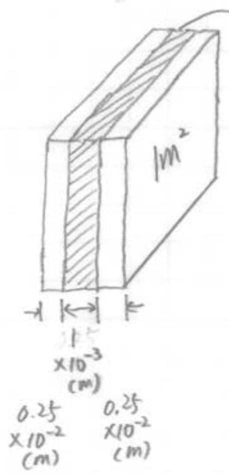
$T_a = 10\,^\circ C ≒ 283\ (K)$

$$\dot{Q}_{total} = \frac{\Delta T}{\frac{L}{kA} + \frac{1}{hA}} \ (ㄱ) = \frac{298 - 283\ (K)}{\frac{0.5\times10^{-2}(m)}{0.75(W/m\cdot K)\times 1(m^2)} + \frac{1}{10(W/m^2\cdot K)\times 1(m^2)}} = 140.625\ (W)$$

$1\ year = 365\ days = 365 \times 24\ (hours) = 8760\ (hours)$

$W_{total} = 0.140625\ (KW) \times 8760\ (hours) = 1231.875\ (KWh)$

Annual Cost : $C_1 = 1231.875\ (KWh) \times 0.15\ (\$/kWh) = 184.78\ (\$)$




Thermal Conductivity of Air : $26.3 \times 10^{-3}\ (W/m \cdot K)$

$$\dot{Q}_{total,2} = \frac{\Delta T}{2\times\frac{L_2}{kA} + \frac{L_2}{k_a A} + \frac{1}{hA}} \ (ㄱ)$$

$$= \frac{298 - 283\ (K)}{2\times\frac{0.25\times10^{-2}(m)}{0.75(W/m\cdot K)\times 1(m^2)} + \frac{1\times10^{-3}\ (m)}{26.3\times10^{-3}(W/m\cdot K)\times 1(m^2)} + \frac{1}{10(W/m^2\cdot K)\times 1(m^2)}}$$

$$≒ 103.67\ (W)$$

$W_{total,2} = 0.10367\ (KW) \times 8760\ (hours) = 908.15\ (kWh)$

Annual Cost : $C_2 = 908.15\ (kWh) \times 0.15\ (\$/kWh) = 136.22\ (\$)$

Annual Savings $= C_1 - C_2 = 48.56\ (\$)$

10.3.

$r_0 = \frac{0.05}{2}(m)$ , $\dot{g} = 8\times10^7 (W/m^3)$ , $T_\infty = 120°C$

$h = 30000 (W/m^2\cdot K)$ $k_{ur} = 30 (W/m\cdot K)$

In cylinderical coordinates, the heat equation in the radial direction can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

convection at the outer surface : $k\frac{dT(r_0)}{dr} = h[T_\infty - T(r_0)]$

thermal symmetry about the centerline : $\frac{dT(0)}{dr} = 0$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{g}}{k}r$$

$$r\frac{dT}{dr} = -\frac{\dot{g}}{k}\frac{1}{2}r^2 + C_1$$

at $r=0$; $0\times\frac{dT(0)}{dr} = -\frac{\dot{g}}{2k}\times0^2 + C_1 \rightarrow C_1 = 0$

$$\frac{dT}{dr} = -\frac{\dot{g}}{2k}r \qquad T(r) = -\frac{\dot{g}}{4k}r^2 + C_2$$

at $r=r_0$ : $\cancel{k}\left(-\frac{\dot{g}}{2\cancel{k}}r_0\right) = h\left[T_\infty + \frac{\dot{g}}{4k}r_0^2 - C_2\right] \rightarrow C_2 = T_\infty + \frac{\dot{g}r_0}{2h} + \frac{\dot{g}r_0^2}{4k}$

Therefore, $T(r) = T_\infty + \frac{\dot{g}}{4k}(r_0^2 - r^2) + \frac{\dot{g}r_0}{2h}$

(6)

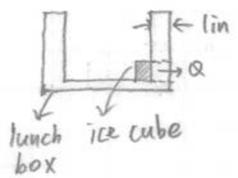
Center Temperature $(r=0)$ : $T(0) = T_\infty + \frac{\dot{g}}{4k}(r_0^2) + \frac{\dot{g}r_0}{2h}$

$$= 120 + \frac{8\times10^7}{4\times30}\times\left(\frac{0.05}{2}\right)^2 + \frac{8\times10^7\times0.025}{2\times3\times10^4} = 570°C$$ (2)

Surface Temperature $(r=r_0)$ : $T(r_0) = T_\infty + \frac{\dot{g}r_0}{2h} = 120 + \frac{8\times10^7\times0.025}{2\times3\times10^4} \doteq 153.3°C$ (2)

10

10.11




Assumption: ① steady operating conditions exist.
② The inner and outer surface temperature of the ice remains 0°C and 32.2°C (90°F)
③ Thermal properties of the ice are at 0°C
④ Heat flux is going through only one side of the ice cube and the lunch box

$k = 0.05 (W/m \cdot K)$
$T_\infty = 90°F (=32.2°C)$
$h = 10 (W/m^2 \cdot K)$

The heat of fusion of ice at 0°C is $h_f = 333.7 (kJ/kg)$

The density of ice is $\rho = 920 (kg/m^3)$

$$A = 1 (in^2) = (0.0254)^2 = 0.000645 (m^2)$$

$$\dot{Q} = kA \frac{\Delta T}{L}_{(3)} = 0.05 \times 0.000645 \times \frac{(32.2-0)}{0.0254}$$

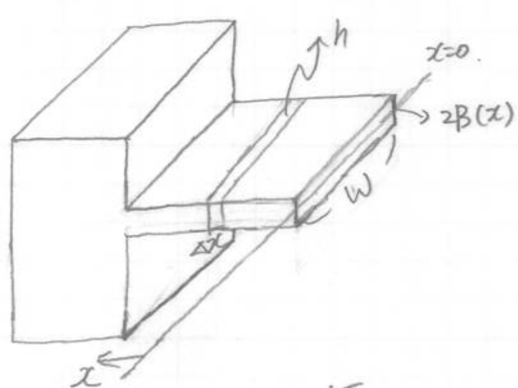
$$= 0.041 (W) \qquad ※ [W] = [J/s]$$

$$Q = m h_f = \rho V h_{f(3)} = 920 \times (0.0254)^3 \times \frac{1}{2} \times 333.7 \times 10^3 \doteqdot 2515 (J)$$

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{2515 (J)}{0.041 (J/s)} = 61341 (sec) = 17 (hour)$$

10

10.17.




$$\begin{cases} T(x) = ax + T_a \\ \frac{dT}{dx} = a \end{cases}$$

conductive flux: $-k \frac{dT}{dx}$

convective flux: $h [T(x) - T_a]$

$$\left[-k\frac{dT}{dx}\Big|_x - \left(-k\frac{dT}{dx}\Big|_{x+\Delta x}\right)\right] \cancel{2}B(x) \cdot \cancel{W} = h [T(x) - T_a] \cancel{2}\cancel{W}(\Delta x) \quad (5)$$

if $\Delta x \to 0$

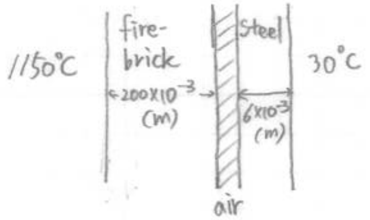
$$B(x) \frac{d}{dx}\Big[k \underbrace{\frac{dT}{dx}}_{a}\Big] = h [\underbrace{T(x) - T_a}_{ax}]$$

$$B(x) \frac{d}{dx}[k \cancel{a}] = h [\cancel{a} x]$$

$$B(x) = \frac{h}{k} \cdot \frac{x^2}{2} \Leftarrow B(x) \text{ is parabolic.}$$

10

10.24



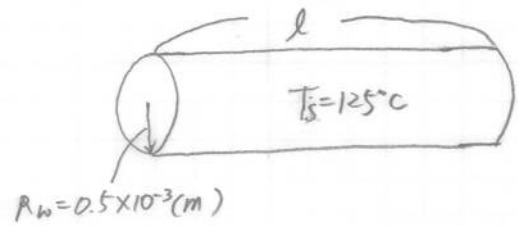

Assume that the steel cladding has seperated from the firebrick, leaving a thin layer of air

$k_F = 1.52 (W/m \cdot °C)$ $k_S = 45 (W/m \cdot °C)$ $k_A = 0.027715 (W/m \cdot °C)$

$$\dot{q} = \frac{\Delta T}{\frac{L_F}{k_F} + \frac{L_A}{k_A} + \frac{L_S}{k_S}} \quad (5) \quad = \frac{1150 - 30\ (°C)}{\frac{200 \times 10^{-3} (m)}{1.52 (W/m \cdot °C)} + \frac{L_A}{0.027715 (W/m \cdot °C)} + \frac{6 \times 10^{-3} (m)}{45 (W/m \cdot °C)}} = 826 (W/m^2)$$

the air film thickness : $L_A \fallingdotseq 0.0339\ (m)$ 10

10.27.




$T_\infty = 25°C$

$\dot{g} = 1.5 (W/m)$, $A_i = 2\pi R_w l\ (m^2)$

(a) $\dot{Q} = \dot{g} l = h A_i \Delta T$ (1)

$$\Rightarrow 1.5 \times \cancel{l} = h \times 2\pi \times (0.5 \times 10^{-3}) \times \cancel{l} \times (125 - 25)$$

$$h = \frac{1.5}{2\pi \times (0.5 \times 10^{-3}) \times 100} \fallingdotseq 4.77 (W/m^2 \cdot K)$$ 2

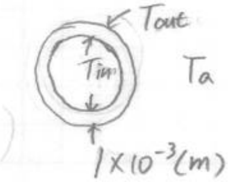
(b)

Because of the steady state,

the rate of heat generation = the rate of heat loss

$$\dot{g} = \frac{\dot{Q}_{cond, cyl}}{l} = 1.5 (W/m)$$ 2

(c) $\dot{Q} = \dot{g}\ell = hA_2\Delta T = hA_2(T_{out} - T_a)$ (2)




$R_o = 1.5\times10^{-3}$ (m)

$A_2 = 2\pi(1.5)\ell\times10^{-3}$ (m)

$$1.5\times\cancel{\ell} = 5\times2\pi\times(0.5+1)\times10^{-3}\times\cancel{\ell}\times(T_{out}-25)$$

$T_{out} \doteqdot 56.8$ (°C)  3

(d) $$\dot{Q} = \dot{g}\cancel{\ell} = \frac{\Delta T}{\frac{1}{hA_2} + \frac{\ln(R_o/R_w)}{2\pi k\ell}} \; (2) = 2\pi\cancel{\ell}\frac{(T_{in}-T_a)}{\frac{1}{hR_o} + \frac{\ln(R_o/R_w)}{k}}$$

$$1.5 = \frac{2\pi(T_{in}-25)}{\frac{1}{5\times1.5\times10^{-3}} + \frac{\ln(1.5/0.5)}{0.25}}$$

$T_{in} \doteqdot 57.9$ (°C)  3