1. Heat transfer under steady-state conditions means that the temperature of a body at any point does not change with time. However, the temperature of a body various with time as well as position in real world. The generation or removal of heat over time causes a part's materials to expand or contact. Such changes produce stress and deflection that may cause otherwise suitable parts to fail. Consequently, we have to analyze transient heat transfer to prevent these causes and this is the reason why transient heat transfer is important. 10

2. Assumption

- The beef is a long homogeneous cylinderical object.
- The thermal properties of the beef is assumed as same as that of water.
- Bi number is much larger than because the temperature distribution within the beef is not even close to being uniform (The outer parts of the beef are well-done while the center part is barely warm)
- Properties: $T_0 \doteqdot 25(^\circ C)$ (initial) $T_a \doteqdot 400(^\circ F) = 205(^\circ C)$ (in oven) $T_f \doteqdot 185(^\circ F) = 85(^\circ C)$ (final) $r_{beef} = 0.1(m)$ (radius)

long cylinder for $Bi \gg 1$

$$X_{Fo} = \frac{\alpha t}{r^2} = \frac{1.4\times10^{-7}\times t}{(0.1)^2}$$

$$\theta = \frac{T_a - T_F}{T_a - T_0} = 0.692 \exp(-5.78\, X_{Fo})$$

$$\frac{205-85}{205-25} = 0.692 \exp\left(-5.78\times \frac{1.4\times10^{-7}\times t}{(0.1)^2}\right)$$

$$t = \frac{(0.1)^2}{1.4\times10^{-7}} \times \frac{1}{(-5.78)} \times \ln\left(\frac{0.667}{0.692}\right) \simeq 454.71(\text{sec}) \simeq 7.58\,(\text{min})$$

(10)

11.1 $D = 1.1\ (cm)$ $L = 7\ (cm)$ $T_0 = 875\ (°C)$ $\alpha = 0.0628\ (cm^2/s)$

$k = 0.146\ (W/cm \cdot K)$ $h = (W/cm^2 \cdot K) = ?$

Time (s)	0	25	50	75	> 200
T (°C)	871	831	810	800	792.

$T_a = 792\ (°C)$

$\alpha = \frac{k}{\rho C_p}$

$$\theta = \frac{T - T_a}{T_0 - T_a} = \exp\left(-\frac{hA}{\rho C_p \cdot V} t\right) \Rightarrow \ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -\frac{hA}{\rho C_p \cdot V} \cdot t \Rightarrow \frac{\ln\left(\frac{T - T_a}{T_0 - T_a}\right)}{t} = -\frac{hA}{\rho C_p \cdot V}$$

(3) (4)

t x	$\ln\left(\frac{T - T_a}{T_0 - T_a}\right)$ y
0	-0.05
25	-0.76
50	-1.53
75	-2.34

$$\Rightarrow \frac{\ln\left(\frac{T - T_a}{T_0 - T_a}\right)}{t} \doteqdot -0.0305$$

$\frac{\pi}{4} D^2 L$

$$0.0305 = \frac{hA}{\rho \cdot C_p \cdot V} \Rightarrow h = 0.0305 \left(\frac{V}{A}\right)\left(\frac{k}{\alpha}\right) = 0.0305\left(\frac{1.1}{4}\right)\left(\frac{0.146}{0.0628}\right)$$

$\pi D L$

$$\doteqdot \underline{0.0195\ (W/cm^2 \cdot K)}$$

10

11.2 $\theta = e^{-\tau}$

* Definition of "τ"

$$\theta = \frac{T - T_a}{T_0 - T_a} = \exp\left(-\frac{hA}{\rho C_p V}t\right) = \exp(-\tau) \Rightarrow \tau = \frac{hA}{\rho \cdot C_p \cdot V}\cdot t = \underbrace{(hA)\cdot\left(\frac{1}{\rho C_p V}\right)}_{\text{thermal-time constant}}\cdot \underset{\text{time}}{t}$$

① (hA): $\frac{1}{hA}$ is the resistance to convection heat transfer

② $\left(\frac{1}{\rho C_p V}\right)$: $\rho C_p V$ is thermal capacitance of the solid

※ $\alpha = \frac{k}{\rho \cdot C_p}$

* Relation with Fo and Bi

① thin slab

$$\tau_{slab} = hA\left(\frac{1}{\rho C_p \cdot V}\right)t = h(\cancel{2}\cancel{l^2})\frac{ht}{\rho\cdot C_p\cdot(\cancel{l^2}\times\cancel{2}r)} = \frac{ht}{\rho C_p \cdot r}$$

$$B_i = \frac{hr}{k},\quad F_o = \frac{\alpha t}{r^2} = \frac{kt}{\rho\cdot C_p\cdot r^2} \Rightarrow B_i\cdot F_o = \frac{h\cancel{r}}{\cancel{k}}\cdot\frac{\cancel{k}t}{\rho C_p r^{\cancel{2}}} = \frac{ht}{\rho C_p r}$$

$\Rightarrow \tau_{slab} = B_i \cdot F_o$

② cylinder

$$\tau_{cylinder} = hA\left(\frac{1}{\rho C_p V}\right)t = h(2\cancel{\pi} \cancel{r} \cancel{l})\frac{1}{\rho\cdot C_p\cdot(\cancel{\pi} r^{\cancel{2}} \cancel{l})}\cdot t = \frac{2ht}{\rho\cdot C_p\cdot r}$$

$\Rightarrow \tau_{cylinder} = 2 B_i \cdot F_o$

③ sphere

$$\tau_{sphere} = hA\left(\frac{1}{\rho\cdot C_p\cdot V}\right)t = h(\cancel{4}\cancel{\pi} \cancel{r^2})\frac{1}{\rho\cdot C_p\cdot\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}}\cdot t = \frac{3ht}{\rho\cdot C_p\cdot r}$$

$\Rightarrow \tau_{sphere} = 3\ B_i \cdot F_o$

11.3. For large Bi, choose $\theta = 0.01$ arbitrarily as thermal equilibrium

i) each area per volume is same.

$$\frac{2\ell^2}{2\ell^2 \cdot r_1} = \frac{2\pi r_2 L}{\pi r_2^2 L} = \frac{4\pi r_3^2}{\frac{4}{3}\pi r_3^3} = C \text{ (const)} \Rightarrow \frac{1}{r_1} = \frac{2}{r_2} = \frac{3}{r_3} = C.$$

slab cylinder sphere.

$r_1 = \frac{1}{c}$, $r_2 = \frac{2}{c}$, $r_3 = \frac{3}{c}$.

① thin slab.

$$\theta = 0.01 = 0.81 \exp(-2.47 X_{Fo})$$

$$X_{Fo} = \frac{\alpha t}{r_1^2} = -\frac{1}{2.47} \ln \frac{0.01}{0.81} = 1.78 \Rightarrow t_{slab} = \frac{1.78 \cdot r_1^2}{\alpha} = \frac{1.78 \left(\frac{1}{c}\right)^2}{\alpha}.$$

② long cylinder

$$\theta = 0.01 = 0.692 \exp(-5.78 X_{Fo})$$

$$X_{Fo} = \frac{\alpha t}{r_2^2} = -\frac{1}{5.78} \ln \frac{0.01}{0.692} = 0.733 \Rightarrow t_{cylinder} = \frac{0.733\, r_2^2}{\alpha} = \frac{2.932 \left(\frac{1}{c}\right)^2}{\alpha}$$

$\frac{2}{c}$

③ sphere

$$\theta = 0.01 = 0.608 \exp(-9.87 X_{Fo})$$

$$X_{Fo} = \frac{\alpha t}{r_3^2} = -\frac{1}{9.87} \ln\left(\frac{0.01}{0.608}\right) = 0.416 \Rightarrow t_{sphere} = \frac{0.416\, r_3^2}{\alpha} = \frac{3.744 \left(\frac{1}{c}\right)^2}{\alpha}$$

$\frac{3}{c}$

$\Rightarrow$ rank order : thin slab — long cylinder — sphere.

ii) $r_1 = r_2 = r_3 = r$.

① thin slab.

$$t_{slab} = \frac{1.78 (r)^2}{\alpha}.$$

② long cylinder

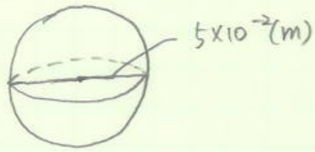
$$t_{cylinder} = \frac{0.733\, r^2}{\alpha}$$

③ sphere

$$t_{sphere} = \frac{0.416\, r^2}{\alpha}.$$

$\Rightarrow$ rank order : sphere — long cylinder — thin slab

11.7



$T_1 = 450\,(°C)$

$T_a = 100\,(°C)$

$h = 1.76\,(Btu/h \cdot ft^2 \cdot °F) = 10\,(W/m^2 \cdot K)$

$T_2 = 150\,(°C)$

$t = ?$ (until $T_1 \to T_2$)

$k = 16\,(W/m \cdot K)$

$\alpha_T = 4.4 \times 10^{-6}\,(m^2/s)$

$V = \frac{4}{3}\pi R^3$

$A = 4\pi R^2$

$\left(\frac{V}{A}\right) = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$

$$Bi = \frac{hR}{k} = \frac{10 \times (2.5 \times 10^{-2})}{16} = 0.0156 \ll 1$$

heat loss is controlled by external convection rather than by internal conduction

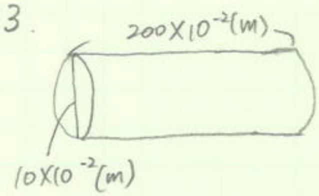
$$\frac{T_2 - T_a}{T_1 - T_a} = \frac{150 - 100}{450 - 100} = \exp\left(\frac{-hAt}{\rho C_p V}\right) = 0.143$$

$$\frac{k}{\rho C_p} = \alpha_T = 4.4 \times 10^{-6} \Rightarrow \rho C_p = \frac{k}{\alpha_T} = \frac{16}{4.4 \times 10^{-6}} = 3636364\,(J/m^3 \cdot K)$$

$$\frac{hAt}{\rho C_p V} = 1.946 \Rightarrow \frac{10 \times 4\pi \times (2.5 \times 10^{-2})^2 \cdot t}{3636364 \times \frac{4}{3}\pi (2.5 \times 10^{-2})^3} = 1.946$$

$t = 5897\,(sec) = 1.64\,(hour)$

11.13.



$k = 14.4\ (W/m \cdot K)$ $\quad T_0 = 38\ (°C)$

$\alpha = 3.9 \times 10^{-6}\ (m^2/s)$ $\quad h = 85\ (W/m^2 \cdot K)$

$T_1 = 593\ (°C)$ $\quad V = \pi R^2 L$

$T_2 = 482\ (°C)$ $\quad A = 2\pi R L$ $\quad t = ?$

$$B_i = \frac{hr}{k} = \frac{85 \times 5 \times 10^{-2}}{14.4} = 0.3.$$

From Fig 11.1.4(e), $B_i = 0.3 \Rightarrow \frac{\theta_1}{\theta_1^\circ} = 0.85$

$$\theta_1 = \frac{482-38}{593-38} = 0.85\,\theta_1^\circ \Rightarrow \theta_1^\circ \doteq 0.94$$

$\lambda_1 = (n\,B_i)^{0.5}$ $\quad n = 2$ cylinder

$= (2 \times 0.3)^{0.5} = \underline{0.77}$

$A_1 = \underline{1.08}$ (from Fig. 11.1.4(a))

$$\theta_1^\circ = A_1 \cdot \exp(-\lambda_1^2 X_{Fo})$$
$$= 1.08 \exp(-(0.77)^2 X_{Fo})$$

$$\Rightarrow X_{Fo} = \frac{\alpha t}{r^2} = -\frac{1}{(0.77)^2} \ln\left(\frac{0.94}{1.08}\right)$$

$$\Rightarrow \underline{t \doteq 150\ (sec) \doteq 2.5\ (min)}$$

11.23.

Bi >> 1 because the sheet of glass is plunged into a stream of running water. In the stream of running water, the effect of convection is much larger than that of conduction.

For Bi >> 1,

$$\theta = \frac{T_a - T_c}{T_a - T_0} = 0.81 \exp(-2.47 X_{Fo})$$

$$= \frac{60-100}{60-300} = 0.17 = 0.81 \exp(-2.47 X_{Fo})$$

$$X_{Fo} = \frac{\alpha t}{r^2} = -\frac{1}{2.47} \ln\left(\frac{0.17}{0.81}\right) = 0.632.$$

$$t = \frac{0.632 r^2}{\alpha} = \frac{0.632\, r^2\, \rho \cdot C_p}{k} \qquad \alpha = \frac{k}{\rho C_p}$$

$$= \frac{0.632 (1/12)^2 \times (155) \times (0.2)}{0.4/3600} = 1224.5\ (\text{sec}) = 20.4\ (\text{min})$$