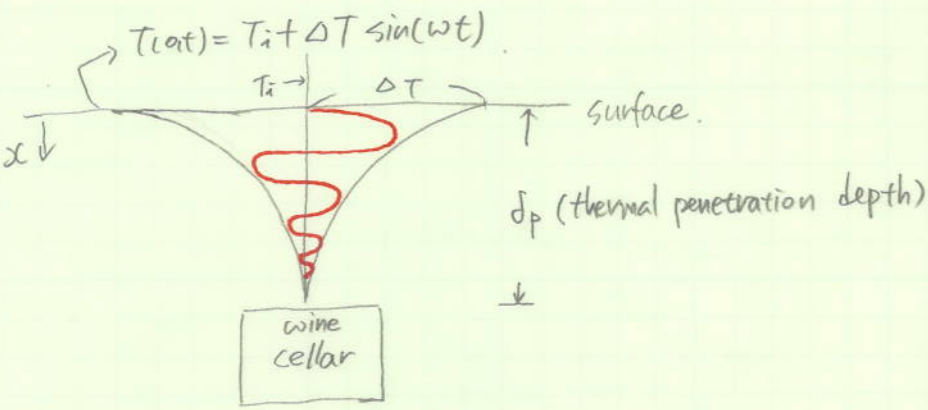
1. In this problem, consider "periodic heating" which occurs naturally in situations involving the collection of solar energy. For a surface temperature history described by $T(0,t) = T_i + \Delta T \sin \omega t$, the solution of $\left(\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \right)$ is

$$" \frac{T(x,t) - T_i}{\Delta t} = \exp\left[-x\sqrt{\omega/2\alpha} \right] \sin\left[\omega t - x\sqrt{\omega/2\alpha} \right] "$$

This solution applies after sufficient time has passed to yield a quasi steady state for which all temperatures fluctuate periodically about a time-invariant value. At locations in the solid, the fluctuations have a "time lag $\left(x\sqrt{\frac{\omega}{2\alpha}} \right)$" relative to surface temperature. Also, the amplitude of the fluctuations within the material decays exponentially with distance from the surface. Consequently, the time when the cellar is coldest is "early spring" because we have to consider the time lag.



2. Heat content is the sum of the internal energy.

$$U = \rho C_p V T = m C_p T \ (kJ/kg \text{ or } J/kg)$$

The heat content of the ocean is much more than that of the atmosphere, so the ocean absorbs and release a larger amount of the internal energy as a huge energy reservior. Consequently, the ocean has greater effects on the temperature of earth than the atmosphere.

11.24 $m=6200(kg)$ $h_1=5861(W/m^2\cdot°C)$ $l=10\times10^{-3}(m)$ $\Delta T_2=60-110(°C)$

$\Delta T_1=20-110\ (°C)$, $h_2=10\times10^3(W/m^2\cdot°C)$ $A=14(m^2)$ $t_{1\to2}=?$

$$Q=m\cdot C_p\cdot\Delta T$$

$$\dot{Q}=\frac{d}{dt}(m\cdot C_p\cdot\Delta T)=-A\frac{\Delta T}{\frac{1}{h_1}+\frac{l}{k}+\frac{1}{h_2}}\Rightarrow\frac{d(\Delta T)}{\Delta T}=-\frac{A}{\left(\frac{1}{h_1}+\frac{l}{k}+\frac{1}{h_2}\right)\cdot m\cdot C_p}t$$

(5)

$$\ln\Delta T_2-\ln\Delta T_1=\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)=-\frac{A}{\left(\frac{1}{h_1}+\frac{l}{k}+\frac{1}{h_2}\right)m\cdot C_p}t$$

$$t=-\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)\times\left(\frac{1}{h_1}+\frac{l}{k}+\frac{1}{h_2}\right)\frac{m\cdot C_p}{A}=-\ln\left(\frac{60-110}{20-110}\right)\times\left(\frac{1}{5861}+\frac{10\times10^{-3}}{16}+\frac{1}{10\times10^3}\right)\times\frac{6200\times4200}{14}$$

$$\simeq 979.2(sec)=16.32(min)$$

11.27

Eq 11.2.15 $-$ $\bar{\theta} = \frac{2A_1 J_1(\lambda_1)}{\lambda_1} \exp(-\lambda_1^2 X_{Fo})$

For $Bi \ll 1$

* $A_1 = 1$ (from Fig 11.1.4(a))

* $\frac{\lambda_1}{(n \cdot Bi)^{1/2}} = 1 \Rightarrow \frac{\lambda_1}{(n \cdot \frac{hr}{k})^{1/2}} = 1 \Rightarrow \lambda_1^2 = n \cdot \frac{hr}{k}$

$X_{Fo} = \frac{\alpha t}{r^2} = \frac{kt}{\rho c_p r^2}$

$-\lambda_1^2 \cdot X_{Fo} = -n \cdot \frac{h\cancel{r}}{\cancel{k}} \times \frac{\cancel{k}t}{\rho c_p r^{\cancel{2}}} = -\frac{nh}{\rho c_p r} t$

① thin slab (n=1) $-\frac{ht}{\rho c_p r} = -\frac{h l^2 t}{\rho c_p (l^2 r)}$ (o) $\left(\begin{array}{l} V = l^2 r \\ A = l^2 \end{array} \right.$

② cylinder (n=2) $-\frac{2ht}{\rho c_p r} = -\frac{h(2\cancel{\pi} \cancel{r} \cancel{l})t}{\rho c_p (\cancel{\pi} r^{\cancel{2}} \cancel{l})} = -\frac{2ht}{\rho c_p r}$ (o) $\left(\begin{array}{l} V = \pi r^2 l \\ A = 2\pi r l \end{array} \right.$

③ sphere (n=3) $-\frac{3ht}{\rho c_p r} = -\frac{h(\cancel{4\pi} r^{\cancel{2}})t}{\rho c_p (\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}})} = -\frac{3ht}{\rho c_p r}$ (o) $\left(\begin{array}{l} V = \frac{4}{3}\pi r^3 \\ A = 4\pi r^2 \end{array} \right.$

* $\frac{J_1(\lambda_1)}{\lambda_1} = \frac{1}{2}$

$\theta = \frac{2A_1 J_1(\lambda_1)}{\lambda_1} \exp(-\lambda_1^2 X_{Fo}) = \exp\left(-\frac{hA}{\rho c_p V} t\right)$ for $Bi \ll 1$

11.28.

$L_S = L \quad L_M = L \times 10^{-2}$ (linear scale factor of 10^{-2})

$\alpha_S = \alpha_M$ (Same materials) $\quad t_M = 5$ (sec) $\quad t_S = ?$

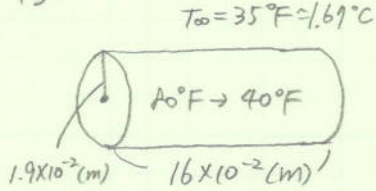
$$(X_{Fo})_S = (X_{Fo})_M$$

$$\Rightarrow \frac{\cancel{\alpha_S} t_S}{L_S^2} = \frac{\cancel{\alpha_M} t_M}{L_M^2} \Rightarrow t_S = \left(\frac{L_S}{L_M}\right)^2 \cancelto{1}{\left(\frac{\alpha_M}{\alpha_S}\right)} t_M$$

$$= \left(\frac{1}{10^{-2}}\right)^2 5 = 50000 \text{ (sec)} \simeq 13.89 \text{ (hr)}$$

The Enterprise can withstand such a high temperature for 13.89 (hr). Therefore, Kirk can proceed the circumnavigation of a period of 10 (hr).

10.30.



$T_1 = 80°F \simeq 26.67°C$ $\quad k = 0.62\ (W/m\cdot K)$

$T_2 = 40°F \simeq 4.44°C.$ $\quad \alpha = 1.46\times10^{-7}\ (m^2/s)$

$h = 180\ (W/m^2\cdot K)$

$$Bi = \frac{hr}{k} = \frac{180\times1.9\times10^{-2}}{0.62} \simeq 5.5 \qquad \theta = \frac{4.44-1.67}{26.67-1.67} = 0.1108$$

In Fig 11.1.4 (e), $\quad Bi = 5.5 \Rightarrow \frac{\theta_1}{\theta_1^\circ} = 0.2 \Rightarrow \theta_1^\circ = \frac{\theta_1}{0.2} = \frac{0.1108}{0.2} = 0.554$ (3)

In Fig 11.1.4 (d), $\quad \theta_1^\circ = 0.554$ & $Bi = 5.5 \Rightarrow X_{Fo} = 1.5 = \frac{\alpha t}{r^2}$

$$t = 1.5\times\frac{r^2}{\alpha} = 1.5\,\frac{(1.9\times10^{-2})^2}{1.46\times10^{-7}} \quad (3)$$

$$= 3709\ (sec) = 1.03\ (hr)$$

(10)

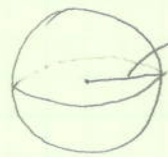
10.32

$$\tilde{y} \equiv \frac{hy}{k} \quad \& \quad \tilde{t} = \frac{h^2 t}{\rho c_p k} = \left(\frac{h}{k}\right)^2 \alpha t \quad \cdots\ (\text{Eq } 11.3.20)$$

$$Bi = \frac{hr}{k} \ \ \&\ \ X_{Fo} = \frac{\alpha t}{r^2} \Rightarrow Bi^2 X_{Fo} = \frac{h^2 \cancel{r^2}}{k^2}\times\frac{\alpha t}{\cancel{r^2}} = \left(\frac{h}{k}\right)^2 \alpha t$$

Therefore, $\tilde{t} = Bi^2\cdot X_{Fo}$.

11.34



$k = 0.6\ (W/m\cdot K)$

$\alpha = 1.4 \times 10^{-7}\ (m^2/s)$

$h = ?$

$$\theta = \frac{T - T_a}{T_0 - T_\infty} = A \cdot e^{-\beta t} \Rightarrow \ln\theta = -\beta t + \ln A \Rightarrow \frac{\ln\theta}{t} = -\beta$$

From Fig P. 11.34

$$\frac{\ln\theta}{t} = -\beta \simeq -0.000094 \Rightarrow \beta = 0.000094$$

$$\beta = \frac{hA}{\rho C_p V} \Rightarrow h = \beta\left(\frac{V}{A}\right)\overset{k/\alpha}{\rho \cdot C_p} = 0.000094\left(\frac{0.1}{3}\right) \times \left(\frac{0.6}{1.4 \times 10^{-7}}\right)$$

$$\simeq 13.43\ (W/m^2\cdot K)$$