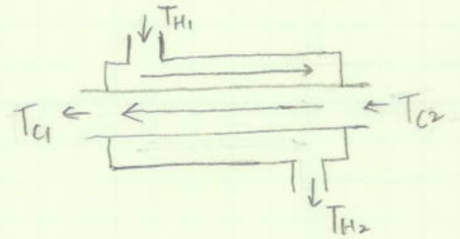
HW #6.

1. Although customers prefer air cooling methods to water cooling methods, many companies want to use water cooling methods. Because the water cooling system is more efficient than the air cooling system, we have to switch to water cooling systems and solve some problems (piping configuration, the delivery of the liquid to local heat exchangers ...) that the water cooling systems have.

2. Arteries to the skin carrying warm blood are intertwined with veins from the skin carrying cold blood, causing the warm arterial blood to exchange heat with the cold venous blood. This reduces the overall heat loss in cold waters. Also, heat exchangers are present in the tongue of baleen whales as large volumes of water flow through their mouths.

(http://en.wikipedia.org/wiki/Heat_exchanger)

13.1. countercurrent double-pipe heat exchanger



$T_{C_1} > T_{C_2}$

$T_{H_1} > T_{H_2}$

i) $dQ_h = (wC_p)_c \, dT_c = C_c \, dT_c$: heat transfer into the cold fluid.

ii) $dQ_h = -(wC_p)_H \, dT_H = -C_H \, dT_H$: heat loss from the exchanger.

$dQ_h = U \, dA \, (T_H - T_c)$: convection & conduction through the surface of pipe wall.

iii) $d(T_H - T_c) = d(\Delta T)$

From i) & ii), $dT_c = \dfrac{dQ_h}{C_c}$ & $dT_H = -\dfrac{dQ_h}{C_H} \Rightarrow \dfrac{C_H}{C_c} = -\dfrac{dT_c}{dT_H} = -\dfrac{T_{C_1} - T_{C_2}}{T_{H_1} - T_{H_2}}$

$$\text{iii)}\ d(\Delta T) = d(T_H - T_c) = dQ_h\left(-\frac{1}{C_H} - \frac{1}{C_c}\right) = -\frac{dQ_h}{C_H}\left(1 + \frac{C_H}{C_c}\right)$$

$$= -\frac{dQ_h}{C_H}\left(\frac{T_{H_1} - T_{H_2} - T_{C_1} + T_{C_2}}{T_{H_1} - T_{H_2}}\right) = +\frac{dQ_h}{C_H}\left[\frac{T_{H_2} - T_{C_2} - (T_{H_1} - T_{C_1})}{T_{H_1} - T_{H_2}}\right]$$

$$= \frac{dQ_h}{Q_h}(\Delta T_2 - \Delta T_1) \quad \begin{bmatrix} \Delta T_2 = T_{H_2} - T_{C_2} \\ \Delta T_1 = T_{H_1} - T_{C_1} \end{bmatrix}$$

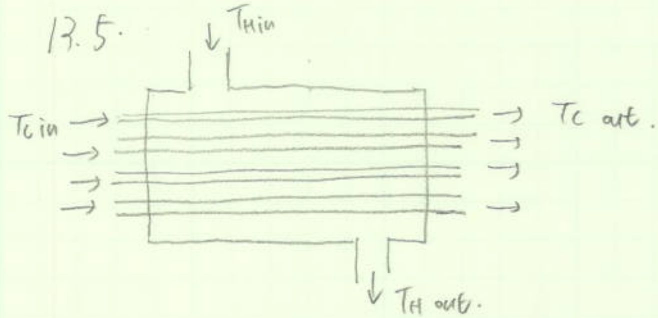
$$= \frac{U \, dA \, \Delta T}{Q_h}(\Delta T_2 - \Delta T_1)$$

$$\Rightarrow \frac{d\Delta T}{\Delta T} = \frac{U}{Q_h}(\Delta T_2 - \Delta T_1) \, dA$$

$$\ln \frac{\Delta T_2}{\Delta T_1} = \frac{U}{Q_h}(\Delta T_2 - \Delta T_1) \, A$$

$$\Rightarrow Q_h = UA \, \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = UA \, \Delta T_{ln} \qquad \left(\Delta T_{ln} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}\right)$$

13.5.



$T_{H\,in} = T_{H}$ out

i th: $C_c[T_{C_z}{}^{i} - T_{C_{z+dz}}] = U \frac{dA}{2}(T_c{}^{i} - T_H)_z \rightarrow \frac{C_c}{U}\frac{dT_c{}^{i}}{dA} = \frac{T_H - T_c{}^{i}}{2}$

define $dn = \frac{U\,dA}{C_c}$

$$\frac{dT_c{}^{i}}{dn} = \frac{T_H - T_c{}^{i}}{2} \Rightarrow \frac{dT_c{}^{i}}{(T_H - T_c{}^{i})} = \frac{dn}{2} \Rightarrow T_c{}^{i} = T_H + a e^{-\frac{n}{2}}$$

use boundary conditions $n=0 \quad T_c = T_{c1} \rightarrow T_H + a = T_{c1}$

$n = n_T \quad T_c = T_{c2}. \rightarrow T_H + a^{-\frac{n_T}{2}} = T_{c2}.$

$$T_{c1} - T_{c2} = a(1 - e^{-\frac{n_T}{2}}) \Rightarrow a = \frac{T_{c1} - T_{c2}}{(1 - e^{-\frac{n_T}{2}})}$$

$$T_c{}^{i} = T_H + \frac{T_{c1} - T_{c2}}{(1 - e^{-\frac{n_T}{2}})} e^{-\frac{n}{2}}$$

✓

$$\frac{C_c}{U}\frac{dT_c{}^{i}}{dA} = \frac{T_H - T_c{}^{i}}{2} \Rightarrow \frac{dT_c{}^{i}}{T_H - T_c{}^{i}} = \frac{U\,dA}{2C_c} \Rightarrow \boxed{\ln\left(\frac{T_H - T_{c1}}{T_H - T_{c2}}\right) = \frac{UA}{2C_c}}$$

✓

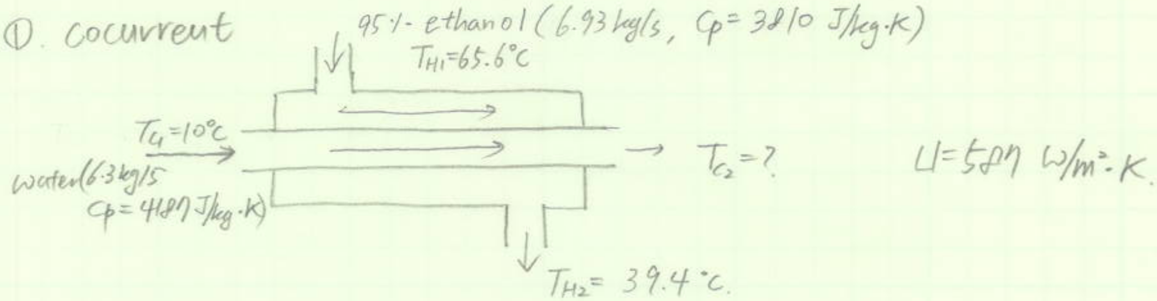
$Q_h = UA\,\Delta T_{ln} \cdot F.$

$$\Rightarrow F = \frac{Q_h}{UA\,\Delta T_{ln}} = \frac{C_c(T_{c2} - T_{c1})}{UA\,\Delta T_{ln}.}$$

P.R

13.17

① cocurrent



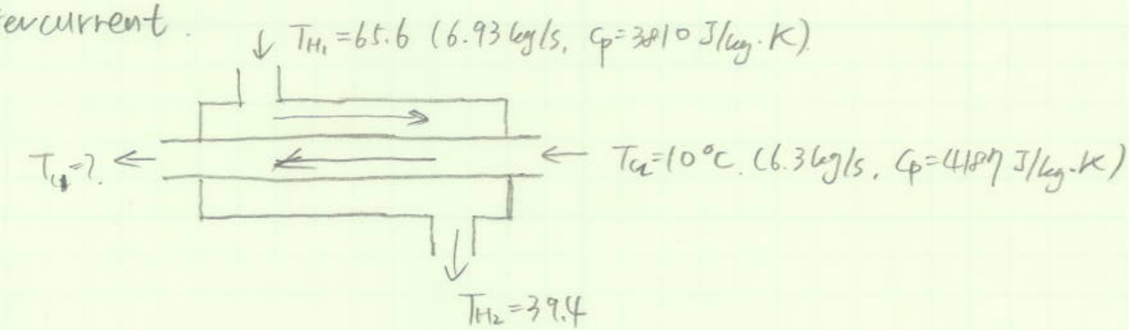
95% ethanol: $\dot{Q} = \dot{m}C_p(T_{H_1} - T_{H_2}) = 691766\ (W)$

water: $\dot{Q} = \dot{m}C_p(T_{C_2} - T_{C_1}) = 691766(W) \Rightarrow T_{C_2} = 10 + \frac{691766}{6.3 \times 4187} = 36.\ ^\circ C$

$\Delta T_1 = T_{H_1} - T_{C_1} = 55.6 \qquad \Delta T_2 = T_{H_2} - T_{C_2} = 3.2 \Rightarrow \Delta T_{ln} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = 18.35$

$\dot{Q} = UA \cdot \Delta T_{ln} \Rightarrow A = \frac{\dot{Q}}{U \cdot \Delta T_{ln}} = \frac{691766}{587 \times 18.35} = 64.22\ (m^2)$

②. countercurrent



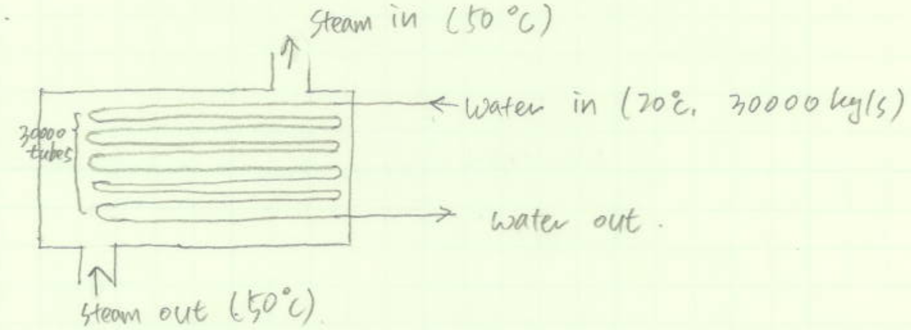
95% ethanol: $\dot{Q} = \dot{m}C_p(T_{H_1} - T_{H_2}) = 691766\ (W)$

water: $\dot{Q} = \dot{m}C_p(T_{C_1} - T_{C_2}) = 691766\ (W) \Rightarrow T_{C_1} = 36^\circ C.$

$\Delta T_1 = T_{H_1} - T_{C_1} = 29.6\ (^\circ C) \quad \Delta T_2 = T_{H_2} - T_{C_2} = 29.4\ ^\circ C \Rightarrow \Delta T_{ln} = 29.5$

$\dot{Q} = UA \cdot \Delta T_{ln} \Rightarrow A = \frac{\dot{Q}}{U \cdot \Delta T_{ln}} = \frac{691766}{587 \times 29.5} = 40\ (m^2)$

13.20.



$q = \dot{m} C_p (T_{c,out} - T_{c,in}) = 2\times10^9 (W) \Rightarrow T_{c,out} = 20 + \dfrac{2\times10^9}{30000\times4182} \simeq 36°C$

$q = UA \cdot \Delta T_{ln} \cdot F \qquad A = N \cdot 2L \cdot (\pi D) \quad (N = 30000)$

$U = \dfrac{1}{\frac{1}{h_{in}} + \frac{1}{h_{out}}}$ h_{out} is given $\Rightarrow$ get h_{in} !!

$Re_D = \dfrac{4\dot{m}}{\pi \cdot D \cdot \mu} = \dfrac{4\times 1 kg/s}{\pi\times0.025\times855\times10^{-6}} = 59567$ turbulent !!

$Nu_D = \dfrac{h_i D}{k} = 0.023\ Re_D^{0.8} \cdot Pr^{0.4} \Rightarrow h_i = Nu_D \dfrac{k}{D} = 7552\ W/m^2 \cdot K$ (?)

$U = \dfrac{1}{\frac{1}{h_{in}} + \frac{1}{h_{out}}} = \dfrac{1}{\frac{1}{7552} + \frac{1}{11000}} = 4478\ (W/m^2 \cdot K)$

$\Delta T_{ln} = \dfrac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln[(T_{h,i} - T_{c,out})/(T_{h,out} - T_{c,in})]} = \dfrac{(50-36) - (50-20)}{\ln(14/30)} = 21°C$

We can get F from Fig. 13.3.2 by P & R.

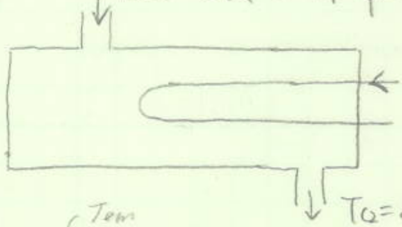
$P = \dfrac{T_{c,out} - T_{c,in}}{T_{H,in} - T_{c,in}} = \dfrac{36-20}{50-20} = 0.53 \qquad R = \dfrac{T_{H,in} - T_{H,out}}{T_{c,in} - T_{c,out}} = 0 \Rightarrow F = 1$!!

$q = U(N \cdot 2L \cdot \pi D) \cdot \Delta T_{ln} \cdot F \Rightarrow L = \dfrac{q}{U(N \cdot 2\pi D)\Delta T_{ln} \cdot F}$

$= \dfrac{2\times10^9}{4478\times(30000\times2\pi\times0.025)\times21\times1}$

$= 4.51\ (m)$

13.25. water (T_{c1}=50°F, c_p=1 Btu/lb·°F)



oil (T_{H1}=200°F, c_p=0.5 Btu/lb·°F, W_{oil}= 5000 lb/h)

UA=488 Btu/h·°F W_{water}=? T_{H2}=?

T_{c2}=80°F

Tem

Assume W= 1950 (lb/h) ⇐ use trial & error method

$Q^{*}_{hc} = (WC_p)_c (T_{c2}-T_{c1}) = 1950 \times 30 = 58500$

$T_{h2} = T_{h1} - \frac{(WC_p)_c}{(WC_p)_h}(T_{c2}-T_{c1}) = 200 - \frac{1950}{2500}(30) = 176.6$

$R = \frac{(WC_p)_c}{(WC_p)_h} = \frac{1950}{2500} = 0.78 \quad P = \frac{T_{c2}-T_{c1}}{T_{H1}-T_{c1}} = \frac{80-50}{200-50} = 0.2 \Rightarrow F = 1$

$Q_h = UA\,\Delta T_{lm}\,F = 488 \times \frac{(200-50)-(176.6-80)}{\ln\left(\frac{150}{96.6}\right)} \times 1 = 59217.84$

$Q^{*}_{hc} = Q_h \Rightarrow W = 1950\,(lb/h),\ T_{h2} = 176.6\ °F$

13.29 $Q_H = (C_p W)_{oil}(T_{c2}-T_{c1}) = (C_p W)_{water}(T_{H1}-T_{H2})$

$\rho_{oil} = 840\ (kg/m^3)$ 800~900

$W_{oil} = \rho Q = 840\,(kg/m^3) \times \frac{1}{1000}\,(m^3/L) \times \frac{1}{1000}\,(L/mL) \times 100\,(mL/s)$

$= 0.084\,(kg/s)$

$T_{H2} = T_{H1} - \frac{(C_p W)_{oil}}{(C_p W)_{water}}(T_{c2}-T_{c1}) =$

$= 100 - \frac{(2000 \times 0.084)}{(4210 \times 0.1)} \times (50-20) = 88.03\ °C.$