HW # VII

10.8

Assume $T_{air} = 25°C$.

(a) $k_{glass} = 0.7 \, (W/m \cdot K)$ $\quad k_{air} = 0.025 \, (W/m \cdot K)$

$Q = UA\Delta T$

$$U_1 = \frac{1}{2 \times \frac{L_{glass}}{k_{glass}} + \frac{L_{air}}{k_{air}}} = \frac{1}{2 \times \frac{0.0025}{0.7} + \frac{0.001}{0.025}} = 21.2 \, (W/m^2 \cdot K)$$

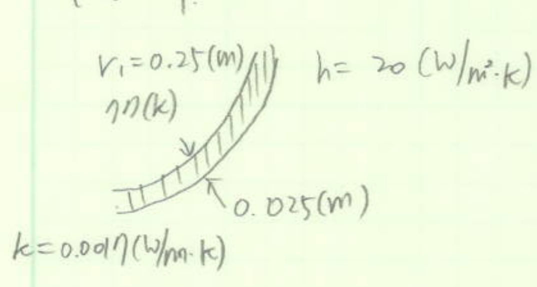
(b)

$$U_2 = \frac{(1 + 0.01)}{2 \times \frac{L_{glass}}{k_{glass}} + \frac{1}{h_{air}}} = \frac{1}{2 \times \frac{L_{glass}}{k_{glass}}}$$

$$\frac{1}{h_{air}} = 2 \times \frac{0.0025}{0.7} - \frac{1}{1.01} \times 2 \times \frac{0.0025}{0.7} = 0.00007071$$

$h_{air} = 14140 \, (W/m^2 \cdot K)$

10.24



$\bar{r}_1 = r_1 + \Delta \frac{1}{2} r = 0.25 + 0.0125 = 0.2625 (m)$

$Q = 4\pi \bar{r}_1^2 U \Delta T$

$$U = \frac{1}{\frac{1}{h} + \frac{\Delta r}{k}} = \frac{1}{\frac{1}{20} + \frac{0.025}{0.0017}}$$

$$= 0.068 \, (W/m^2 \cdot K)$$

$Q = 4\pi (0.2625)^2 (0.068)(300 - 77) = 13 \, (W)$

$$\dot{m} = \frac{13 (W)}{2 \times 10^5 (J/kg)} = 0.000065 \, (kg/s) = 5.6 \, (kg/day)$$

1(day) = 86400 (sec)

11.18

the copper body is "thick" (semi-infinite medium)

$k_{copper} = 401$ (W/m·K) $\quad (\alpha_T)_{copper} = 117\times10^{-6}$ (m²/s)

From (eq. 11.3.20)

$$\tilde{t} = \left(\frac{h}{k_T}\right)^2 \alpha_T t = \left(\frac{100}{401}\right)^2 \times 117\times10^{-6} t = 7.3\times10^{-6} t.$$

Get the surface temperature by using Fig 11.3.2 ($\theta - \tilde{t}^{1/2}$).

t[s]	$\tilde{t}$	$\theta\ (= \frac{T-30}{120-30})$	T[°C]
2000	0.0146	0.88	109.2
8000	0.0584	0.78	100.2
20000	0.146	0.68	91.2
100000	0.73	0.46	71.4
10^6	7.3	0.06	35.4

$t = 10^6$ (sec)

$$\delta_T = 4\sqrt{\alpha_T t} = 4\sqrt{117\times10^{-6}\times10^6} = 43.27$$

thickness $= 2\times\delta_T = 83.54$ (m)

11.29. $\theta_1 = A_1 e^{-\lambda_1^2 X_{Fo}} \cos \lambda_1 \xi$

$$\bar{\theta}_1 = \int_0^1 \theta_1(\xi, t)\, d\xi = A_1 e^{-\lambda_1^2 X_{Fo}} \int_0^1 \cos \lambda_1 \xi \, d\xi$$

$$= \frac{A_1}{\lambda_1} \sin \lambda_1 \, e^{-\lambda_1^2 X_{Fo}}$$

For $Bi = 8$, from Fig. 11.1.4(a)

$A_1 = 1.24$ and $\lambda_1/\sqrt{8} = 0.46$

$\sin \lambda_1 = \sin 1.3 = 0.964$

$$\bar{\theta}_1 = \frac{1.24}{1.3}(0.964)\, e^{-1.69 X_{Fo}} = 0.92\, e^{-1.69 X_{Fo}}$$

12.7. $\theta = \frac{T_2 - T_R}{T_1 - T_R} = \exp\left(-4 \times St \times \frac{L}{D}\right)$

$St = \frac{\overline{Nu}}{Re \cdot Pr}$

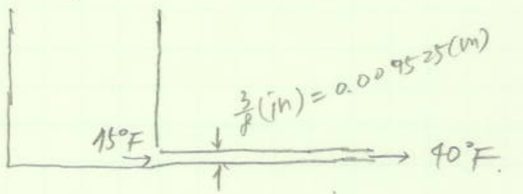
$Re \cdot Pr \cdot \frac{D}{L} = \frac{40}{\pi}(6)\frac{1}{60} = \frac{4}{\pi}$ $\quad \overline{Nu} = 3.66$ (from Fig 12.5.6)

$St = \frac{3.66}{\frac{40}{\pi}(6)} = 0.048$

$\theta = \exp(-4 \times 0.048 \times 60) = e^{-11.5} = 10^{-5}$.

$T_2 = 90 + (50-90) \times 10^{-5} \simeq \underline{90\,°C}$.

12.62.



$T_1 = 75\,(°F)$ $\quad T_2 = 40\,°(F)$

$\dot{Q} = 2\,(gal/min) = \frac{0.007571}{60}\,(m^3/s)$ $\quad P_r = 8.09$

$\rho = 1000\,(kg/m^3)$ $\quad \mu = 1.138 \times 10^{-3}\,(kg/m \cdot s)$

$Re = \frac{4\omega}{\pi \cdot D \cdot \mu}$ ($\omega \rightarrow \rho\dot{Q}$) $= \frac{4 \times 1000 \times 0.007571/60}{\pi \times 0.009525 \times 1.138 \times 10^{-3}} = 14822$ (turbulent)

$f = 0.08\,Re^{-0.25} = 0.00725$ $\quad j_H = f/2 = 0.0036$ $\quad P_r^{2/3} = 4.03$.

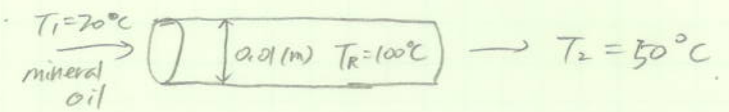
$St = \frac{J_H}{P_r^{2/3}} = 0.0008933$.

$\theta = \frac{T_2 - T_R}{T_1 - T_R} = \exp\left(-4 \cdot St \cdot \frac{L}{D}\right)$

$= \frac{40-32}{75-32} = \exp\left(-4 \times 0.0008933 \times \frac{L}{0.009525}\right) = 0.186$.

$\Rightarrow L = -\frac{1}{4} \times \frac{1}{0.0008933} \times 0.009525 \times \ln(0.186) = \underline{4.67\,(m)}$

13.12



$\dot{Q} = 0.01 (l/s)$
$C_p = 2000 (J/kg \cdot K)$
$k = 0.147 (W/m \cdot K)$
$\rho = 830 (kg/m^3)$

the mean of the wall temperature and the oil temperature : $T_f = \frac{1}{2}(35 + 100) = 67°C$.

At 67°C, $\mu = 0.076 \exp(-0.0384\, T) = 0.0058\ (kg/m \cdot s)$

$$\theta = \frac{T_2 - T_R}{T_1 - T_R} = \frac{50 - 100}{20 - 100} = 0.625.$$

$$Pr = \frac{\mu C_p}{k} = \frac{0.058 \times 2000}{0.147} = 79$$

$$Re = \frac{4w}{\pi \cdot D \cdot \mu} = \frac{4 \times 0.0001 \times 830}{\pi \times 0.01 \times 0.0058} = 182.$$

$$z^* = L^* = \frac{\alpha L}{UR^2} = \frac{4\, L/D}{Pr \cdot Re_D} \approx 0.09 \Rightarrow \frac{L}{D} = \frac{0.09\, Re \cdot Pr}{4} = 321$$

$L = 321 \times 0.01 = 3.21\ (m)$: a rough estimate

$$\overline{Nu}_D = 1.6\, (Re_D \cdot Pr \cdot D/L)^{1/3} \quad (\text{eq } 12.5.49)$$

$$\overline{St} \cdot \frac{L}{D} = \frac{\overline{Nu}}{Re_D \cdot Pr \cdot D/L} = 1.6\, (Re_D \cdot Pr \cdot D/L)^{-2/3}$$

$$\theta = 0.625 = \exp\left[-4 \cdot 1.6\, (Re_D \cdot Pr \cdot D/L)^{-2/3}\right] \Rightarrow Re \cdot Pr \cdot D/L = 50.$$

$L/D = Re \cdot Pr / 50 = 286$. $\quad L = 2.86\ (m)$

$$\bar{h} = \frac{k \cdot \overline{Nu}}{D} = \frac{0.147\,(5.9)}{0.01} = 86.5\ (W/m^2 \cdot K)$$

13.23.
$$4 \cdot St\, \frac{L}{D} = \frac{\pi D L \cdot h}{w \cdot C_p} = \frac{hA}{w \cdot C_p} \equiv \frac{UA}{w \cdot C_p}$$

$$\Rightarrow St \equiv \frac{D}{4L} \frac{UA}{w \cdot C_p} = \frac{D}{4L} NTU$$