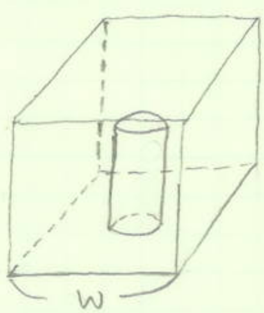
1. The principle of displacement ventilation involves air supply and distribution in a room by upwards displacement. It purports to use natural convection to provide better air quality than convectional "mixing" ventilation. Cool air is introduced in the lower half of the room while warm air exists through an outlet in the celling. This air flow pattern differs greatly from that caused by convectional mixing supply jets. The newly supplied air is slightly coler than the air in the room, and thus has a strong tendency to fall and spread out over the floor in a uniformly thin layer (approximately 20 cm), due to gravity, without mixing significantly with the room air above. This process leads to a continual upwards uniform displacement of air in the room.

— from AIVC (Air Infiltration & Ventilation Centre)

2. buoyancy-induced flow in a confined region



assume a sitting man as a cylinder. (L(m) & r(m)) $\Rightarrow w - 2r = b(m)$

$T_f = \frac{T_{man} + T_{air}}{2}$ $\quad \Delta T = T_{man} - T_{air}$ $\quad \beta = \frac{1}{T_f}$

ν_{air} (at T_f) $= \frac{\mu_{air}}{\rho_{air}}$

$$Gr = \left(\frac{g\beta}{\nu_{air}^2} \right) L^3 \, \Delta T \qquad \text{(eq 14.1.8)}$$

$$V_{z\,max} \times \frac{b \rho_{air}}{\mu} = 0.032\, Gr. \qquad \text{(eq 14.2.17)}$$

$$\Rightarrow V_{z\,max} = 0.032\, Gr \times \frac{\mu}{b \rho_{air}}$$

14.6. For an ideal gas,

$p v = RT$

$\Rightarrow \frac{P}{\rho} = \frac{R_G}{M_w} \cdot T \Rightarrow \rho = \frac{P \cdot M_w}{R_G \cdot T} = \frac{a}{T}$ at constant pressure

$(R_G = 8.3145 \ kJ/kmol \cdot K)$.

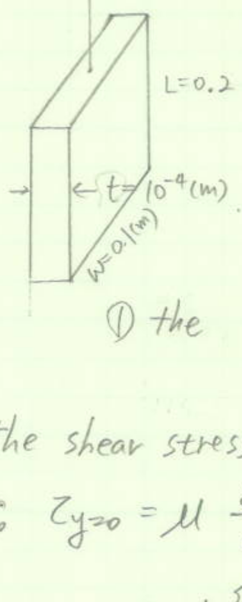
$\Rightarrow \frac{\partial \rho}{\partial T} = -\frac{a}{T^2}$

(eq 14.1.2) $\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$

$\Rightarrow \beta = -\frac{T}{a}\left(-\frac{a}{T^2}\right) = \frac{1}{T}$

$\Rightarrow \beta = \frac{1}{T}$

14.11



① velocity profile ⇒ ② local shear stress ⇒ ③ shear force.

$T_f = \frac{1}{2}(150+50) = 100°C = 373(K)$

$\rho_{air} = 0.9 (kg/m^3) \qquad \beta = \frac{1}{T_f} = 0.0027 (K^{-1})$

$\mu_{air} = 2.1 \times 10^{-5} (Pa \cdot s) \qquad P_r = 0.7$

① the velocity profile : $V_z = V(z) \frac{y}{\delta(z)}\left[1 - \frac{y}{\delta(z)}\right]^2$ (eq 14.7.8)

② the shear stress on the plate (one side)

: $\tau_{y=0} = \mu \left.\frac{\partial V_z}{\partial y}\right|_{y=0}$ (eq 14.7.3)

$= \mu \left\{ \frac{V(z)}{\delta(z)}\left[1 - \frac{y}{\delta(z)^2}\right]^2 + \frac{V(z)}{\delta(z)} y(2)\left(1 - \frac{y}{\delta(z)}\right)\left(-\frac{1}{\delta(z)}\right)\right\}_{y=0}$

$= \mu \frac{V(z)}{\delta(z)}$

(eq. 14.7.15) $V(z) = 5.17 \left(P_r + \frac{20}{21}\right)^{-1/2} \left[g \cdot \beta (T_0 - T_1) z\right]^{1/2}$

$= 5.17 \left(0.7 + \frac{20}{21}\right)^{-1/2} \left[9.8 \times (0.0027)(150-50) z\right]^{1/2}$

$= 6.5 \ z^{1/2}$

(eq. 14.7.16) $\delta(z) = 3.93 \left(P_r + \frac{20}{21}\right)^{1/4} \left[\frac{\nu^2 z}{g\beta(T_0 - T_1)}\right]^{1/4}$

$= 3.93 \left(0.7 + \frac{20}{21}\right)^{1/4} \left[\frac{(2 \times 10^{-5}/0.9)^2 z}{9.8(0.0027)(150-50)}\right]^{1/4}$

$= 0.016 \ z^{1/4}$

$= \mu \left(\frac{6.5}{0.016}\right) z^{1/4}$

$= 8.5 \times 10^{-3} z^{1/4} (Pa)$

③ the shear force :

$$F = 2W\int_0^L \tau_{y=0}\, dz = 2(0.1)(8.5\times10^{-3})\,\frac{4}{5}\,L^{5/4} \quad (L=0.2)$$

$$= 1.8\times10^{-4}\,(N)$$

the weight of the sheet :

$$mg = g\cdot\rho\cdot(W\cdot L\cdot t) = 9.8(6000)(0.1\times0.2\times10^{-4}) = 0.1176\,(N).$$

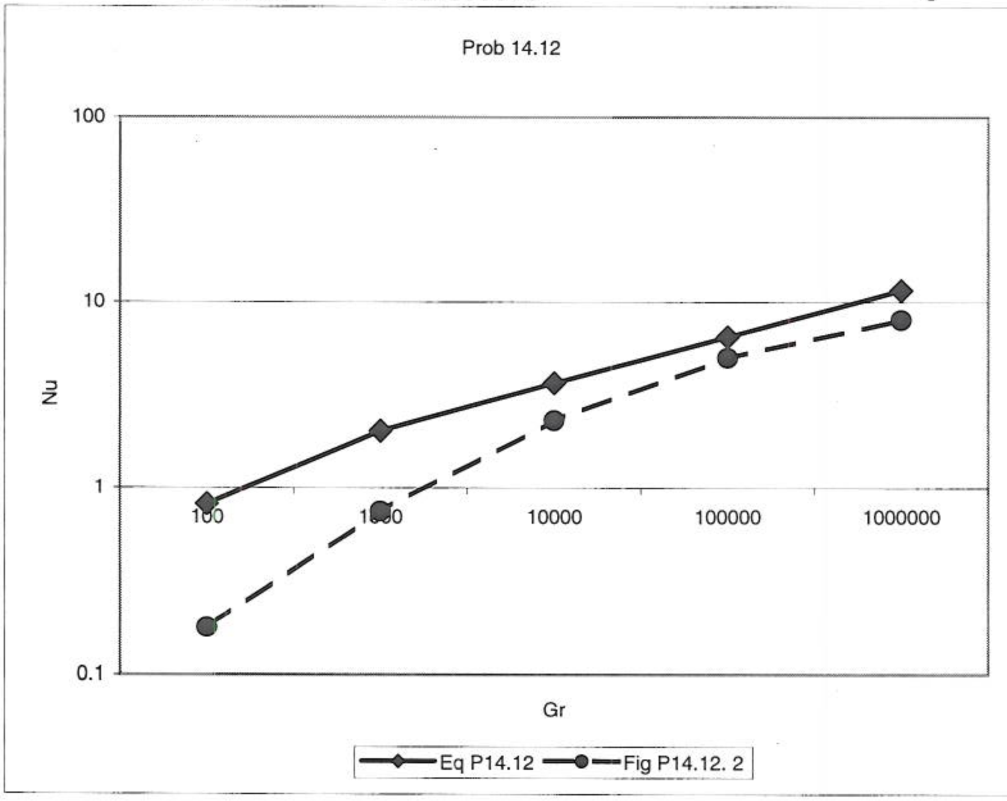
weight >> shear force ⇒ the shear stress cannot lift the sheet!!

14.12.

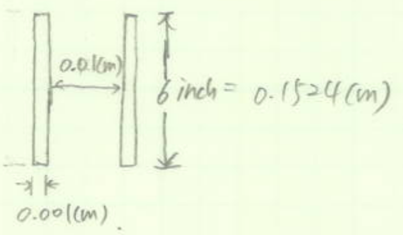
For $Pr=0.7$, Eq. P.14.12 is $\overline{Nu} = \left(\frac{7350}{Gr^2} + \frac{11.5}{Gr^{1/2}}\right)^{-1/2}$

This equation gives results quite different for $Gr<10^4$. This part of the discrepancy is a reflection of the effect of G/W on the flow and on the heat transfer. We can expect that a small G/W chokes or suppresses the flow, and so reduces the heat transfer. At low Gr, the effect of G/W<1 is to lower Nu by a significant factor, in comparison to an "open" (G/W=1) channel.

Gr	*Eq P14.12*	*Fig P14.12.2*
100	0.82060994	0.18
1000	2.022283667	0.75
10000	3.649695804	2.3
100000	6.493257696	5
1000000	11.54699973	8



14.16.



silicon : $k_T = 153$ (W/m·K)

$\rho = 2300$ (kg/m³)

$C_p = 700$ (J/kg·K)

air : $\rho = 0.75$ (kg/m³)

$C_p = 1000$ (J/kg·K)

$k_T = 0.0035$ (W/m·K)

$\mu = 2.2\times10^{-5}$ (Pa·s)

$\beta = \frac{1}{433} = 2.3\times10^{-3} (K^{-1})$

$T_f = \frac{1}{2}(300+25) = 160°C = 433 (K)$

Assume that Bi is small. !! $\frac{d}{dt}\left[\rho C_p \cdot V \cdot T_s\right] = -h\cdot A(T_s - T_a)$

b/L is small => use (eqn 14.8.15) !!

$$Nu = \left(\frac{576}{Ra'^2} + \frac{3.3}{Ra'^{1/2}}\right)^{-1/2} \qquad \left(Ra' = \frac{\rho^2 g\beta b^3 C_p (T_w - T_0)}{\mu\cdot k}\cdot\frac{b}{L}\right.$$

$$= \frac{0.75^2\cdot(9.8)(2.3\times10^{-3})(0.01)^4\cdot10^3}{2.2\times10^{-5}\cdot(0.035)(0.15)}(T_s - T_a)$$

$$\left.= 1.1\,(T_s - T_a)\right)$$

i) $T_s - T_a = 275°C$ ($T_s = 300°C$, $T_a = 25°C$) initial state.

$$Ra' = 1.1(275) = 300$$

$$Nu_1 = \left[\left(\frac{576}{300^2}\right) + \left(\frac{3.3}{(300)^{1/2}}\right)\right]^{-1/2} = (0.0064 + 0.19)^{-1/2} = 2.3$$

ii) $T_s - T_a = 5°C$ (Assume $T_f = 40°C = 313 (K)$)

$$Ra' = \frac{(1)^2(9.8)(3.2\times10^{-3})\,0.01^4\times10^3}{2\times10^{-5}\times(0.025)(0.15)}(5) = 21$$

$$Nu_2 = \left[\left(\frac{576}{(21)^2}\right) + \left(\frac{3.3}{(21)^{1/2}}\right)\right]^{-1/2} = (1.3 + 0.72)^{-1/2} = 0.7$$

$$\frac{Nu_1}{Nu_2} \simeq 3.3 \qquad \frac{[576/(21)^2]}{[576/(300)^2]} \simeq 204 \qquad \frac{[3.3/(21)^{1/2}]}{[3.3/(300)^{1/2}]} = 3.78$$

$\Rightarrow$ Nu is dominated by the term $\left[3.3/(Ra')^{1/2}\right]$

$$\Rightarrow Nu = \left(\cancel{\frac{576}{Ra'^2}} + \frac{3.3}{Ra'^{1/2}}\right)^{-1/2} = 0.55(Ra')^{1/4} = 0.55\left[1.1(T_s - T_a)\right]^{1/4}$$

$$= 0.56\,(T_s - T_a)^{1/4}$$

$$h = Nu\cdot\frac{k_{air}}{b} = \frac{0.035}{0.01}Nu = 3.5\,Nu = 3.5\times0.56\,(T_s - T_a)^{1/4}$$

$$\frac{dT_s}{dt} = -h\,\frac{(A/V)}{\rho \cdot C_p}(T_s - T_a) = -3.5\,\frac{(2000)}{2300 \times 700} \cdot 0.56\,(T_s - T_a)^{5/4}.$$

$$= -2.4 \times 10^{-3}(T_s - T_a)^{5/4}.$$

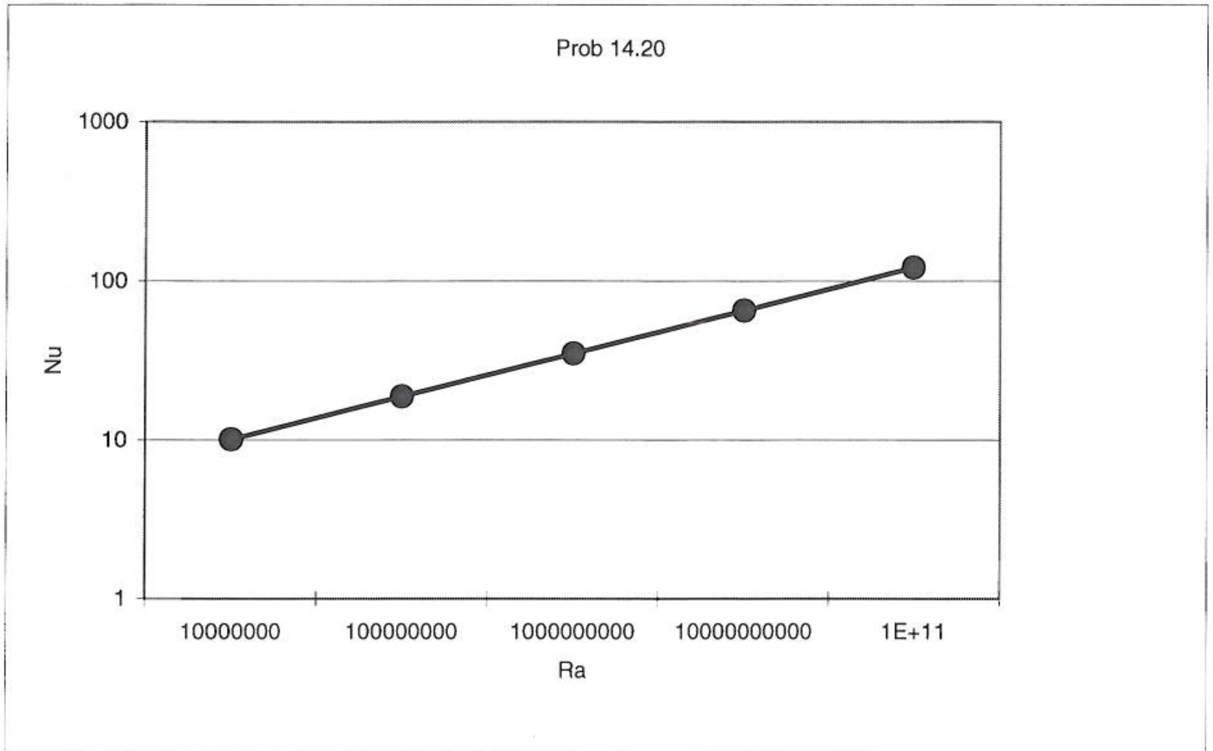
$$\Rightarrow \frac{d(T_s - T_a)}{(T_s - T_a)^{5/4}} = 2.4 \times 10^{-3}\,t.$$

$$\Rightarrow (T_s - T_a)^{-1/4} = 0.246 + 6 \times 10^{-4}\,t. \quad \left(\text{using } T_s - T_a = 275 \text{ at } t = 0. \ \text{(initial condition)}\right)$$

$$\Rightarrow T_s - T_a = (0.246 + 6 \times 10^{-4}\,t)^{-4}$$

plot the relationship between $(T_s - T_a)$ & t.

$t \simeq 10^3$ (sec)

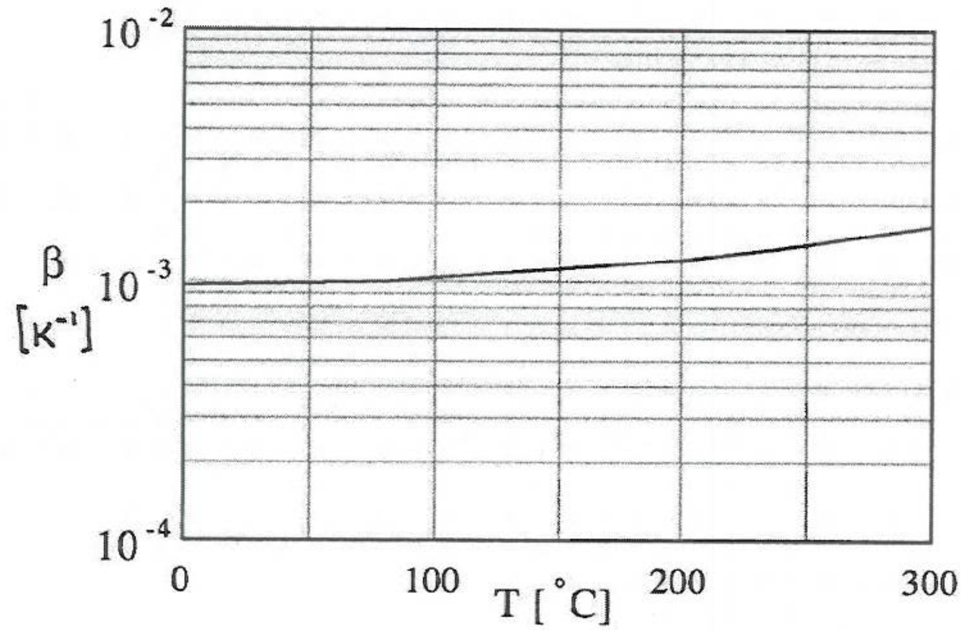


Ra	*Nu*
10000000	10.09121
1E+08	18.79072
1E+09	34.98995
1E+10	65.15434
1E+11	121.3231

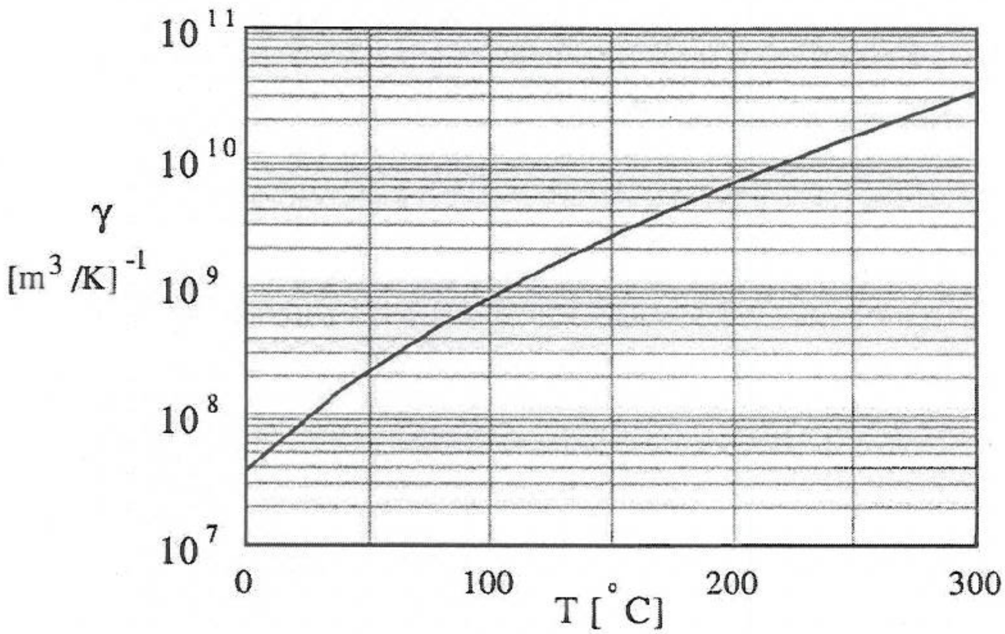
Nu = 0.17 (Ra^0.23)

C=0.17 n=0.23

14.24



10 -2
β
[K-1]
10 -3
10 -4
0
100
200
300
T [°C]



10 11
10 10
γ
[m3 /K] -1
10 9
10 8
10 7
0
100
200
300
T [°C]