Pipe flow review

Fully developed flow $\Rightarrow \frac{\partial}{\partial x} = 0$, $\frac{\partial p}{\partial x} = \text{const.}$

Dimensional analysis: for flow in pipe

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right) = \frac{\ell}{D} \underbrace{\phi\left(Re, \frac{\varepsilon}{D}\right)}_{f}$$

Laminar: $Q = \frac{\pi D^4 \Delta p}{128 \mu \ell} \rightarrow f = \frac{64}{Re}$

Energy equation: $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$

pump head $(\dot{W} = \gamma Q h_p)$

h_L major $= f \frac{\ell}{D} \frac{V^2}{2g}$ Darcy - Weisbach

Moody chart

h_L minor $\quad h_{L\,minor} = K_L \frac{V^2}{2g}$ look up K_L.

Noncircular: hydraulic diameter $D_h = \frac{4A}{P}$.

Single pipes: Find $\overset{(I)}{\Delta p}$, $\overset{II}{V \text{ or } Q}$, $\overset{III}{D}$.

Multiple pipes. series ΣQ, Σh_L. $\parallel$: ΣQ, h. Reverse?

Pipe flowrate

Ex 8.80 : $p_1 = 0$, $p_2 = 180 \frac{lb}{in^2}$, $V_1 = 0$, $V = V_2 = \frac{Q}{\frac{\pi}{4}D^2} = 5.30 \frac{ft}{s}$

Table 8.1 $\frac{\varepsilon}{D} = \frac{0.00015 \, ft}{(3/12) ft} = 6 \times 10^{-4}$ $Re = \frac{VD}{\nu} = \frac{(5.30 \frac{ft}{s})(\frac{3}{12} ft)}{1.66 \times 10^{-5} \frac{ft^2}{s}} = 7.96 \times 10^4$

$\rightarrow f = 0.0212$ (Fig 8.20)

$h_p = \frac{p_2}{\gamma} + z_2 - z_1 + \left(1 + \frac{f\ell}{D}\right)\frac{V^2}{2g} = 827 ft$ $1 \, hp = 550 \frac{ft \cdot lb}{s}$

$P = \gamma Q h_p = \left(62.4 \frac{lb}{ft^3}\right)\left(0.26 \frac{ft^3}{s}\right)(827 ft) = 24.4 \, hp$

Ex 8.90 Usual eq $p_1 = 150 \, kPa$, $p_2 = 0$

$z_1 = 0.8 m$, $z_2 = \ell \sin 60° = 1.22 m$, $V_1 = 0$

$V = \frac{Q}{A}$ $V_2 = \frac{Q}{A_2} = \frac{A}{A_2} V = \left(\frac{D}{D_2}\right)^2 V = \left(\frac{15 mm}{7.5 mm}\right)^2 V = 4V$

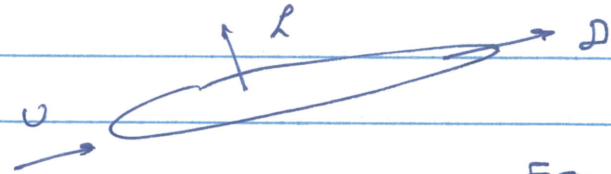
$f = 0.11$, $K_L = 0.75$

$$\frac{150 \times 10^3 \, N/m^2}{9.80 \times 10^3 \, N/m^3} + 0.8 m = 1.22 m + \left[4^2 + 0.11\left(\frac{1.5 m}{0.015 m}\right) + 0.75\right]\frac{V^2}{2(9.81)}$$

i.e. $V = 3.09 \, m/s$

and $Q = AV = \frac{\pi}{4}(0.015 m)^2 (3.09 m/s) = 5.46 \times 10^{-4} \frac{m^3}{s}$

Flow over Immersed Bodies Review



Drag D

Lift L

From skin friction and $\int$ pressure

Re $\gg 1$: narrow region of adjustment from no slip to free stream. Boundary layer (laminar $\rightarrow$ turbulent)

Flat plate: $\partial p/\partial x = 0$ - Blasius solution $\delta \propto \left(\frac{\nu x}{U}\right)^{\frac{1}{2}}$

Similarity solution.

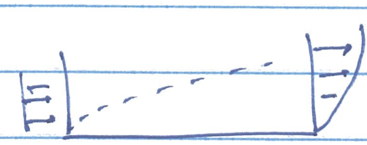
$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \qquad \frac{\delta_*}{x} = \frac{1.721}{\sqrt{Re_x}}$$

Thickness   Displacement thickness

Wall shear stress $\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0.332\, U^{3/2} \sqrt{\frac{\rho\mu}{x}}$

Force $F = b\int_0^{\ell} \tau_w \, dx = 1.328 \left(\frac{1}{2}\rho U^2\right) \frac{b\ell}{\sqrt{Re}} \rightarrow C_D = \frac{1.328}{\sqrt{Re}}$

Momentum integral



$\tau_w = \rho U^2 \frac{d\theta}{dx} = \mu \frac{du}{dy}$. Specify $\frac{u}{U} = g\left(\frac{y}{\delta}\right)$

Can generalize to turbulent case $\tau_w = 0.0225\, \rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$

Moody-type chart.

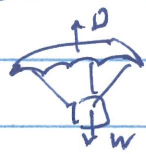
Coefficients : $C_D = \dfrac{D}{\frac{1}{2}\rho U^2 A}$ $\quad C_L = \dfrac{L}{\frac{1}{2}\rho U^2 A}$

A : planform area (drag) $A = bc$

or wing area (lift) Wing loading $\dfrac{L}{A}$

Circulation - can change lift

Ex 9.37 Use properties at sea level $W = D$ $\quad D = 5\,m$, $U = 3\,m/s$, $W = 200\,N$



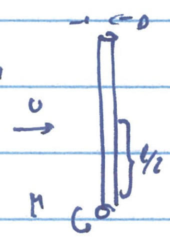
$$C_D = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{200\ N}{\frac{1}{2}(1.23\ kg\ m^{-3})(3\ m/s)^2 \frac{\pi}{4}(5\ m)^2} = 1.84$$

Ex: 9.64

$l = 20$

$D = 0.12$

$U = 20\ m/s$



Eq: $M = \dfrac{l}{2} D$ $\quad Re = \dfrac{UD}{\nu} = \dfrac{(20)(0.12)}{1.46 \times 10^{-5}} = 1.64 \times 10^5$

Fig 9.21 → $C_D = 1.2$ $\quad D = \frac{1}{2}\rho U^2 l D C_D$

$$M = \frac{20}{2} \times 1.2 \times \left(\tfrac{1}{2}\right) \times (1.23) \times (20)^2 \times 20 \times 0.12 = 7080\ N.m$$

Ex 9.104 $A = 200\,ft^2$, $W = 2000\,lb$, $C_L = 0.40$ $\quad C_D = 0.05$

Power for level flight?

$L = W = C_L \frac{1}{2}\rho U^2 A$ $\quad$ ie $2000\ lb = \frac{1}{2}(0.40)\left(0.00238\ \frac{slugs}{ft^3}\right) U^2 (200\ ft^2)$

→ $U = 145\ ft/s$.

$P = DU$ $\quad D = C_D \frac{1}{2}\rho U^2 A$ → $D = W \times \dfrac{C_D}{C_L} = 2000 \times \dfrac{0.05}{0.4} = 250\ lb$

$$P = (250\ lb)\left(145\ \frac{ft}{s}\right) = 3.63 \times 10^4\ \frac{ft\ lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\ lb}{s}}\right) = 65.9\ hp$$

Compressible Flow Review

Ideal gas: $p = RT\rho$

Internal energy $\check{u}_2 - \check{u}_1 = c_v (T_2 - T_1)$ $\quad R = c_p - c_v$

Enthalpy $\check{h}_2 - \check{h}_1 = c_p (T_2 - T_1)$ $\quad k = \frac{c_p}{c_v}$

$ds = 0 \Rightarrow p \propto \rho^k$.

Speed of sound $c^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$ $\quad Ma = \frac{V}{c}$

$= \sqrt{kRT}$ IG

Regimes: subsonic transonic

supersonic Mach cone ~~Ma~~ $= \frac{1}{\sin\alpha}$

Duct flow

ALL . Mass $\dot{m} = \rho A V = $ const

ALL . Energy $d\check{h} + V dV = \delta q$

I, F, N $\rightarrow \check{h} + \frac{1}{2} V^2 = $ const no heat addition

. stagnation values: reduce to $V=0$ isentropically

I, F, N $\delta q = 0$: $\frac{T}{T_0} = \frac{1}{1 + \frac{k-1}{2} Ma^2}$

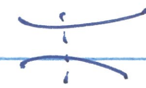
F(ALL) . Momentum $-dp - \frac{\tau_w \pi D}{A} dx = \rho V dV$

R, N $\qquad \tau_w = 0 \quad \Rightarrow \quad p + \rho V^2 = const$

I $\qquad$ OR $\quad dp + \frac{1}{2}\rho V^2 = 0 \quad$ Bernoulli

Isentropic

Ma = 1 only at throat



$dA = 0$: nothing

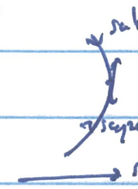
Eqs for $\frac{T}{T_0}$, $\frac{p}{p_0}$, $\frac{\rho}{\rho_0}$

* state sonic: $\frac{p^*}{p_0}$, $\frac{\rho^*}{\rho_0}$, $\frac{T^*}{T_0}$, $\frac{A}{A^*}$ $\qquad$ D.1

Fanno

$\delta q = 0$, $\tau_w \neq 0 \qquad f = \frac{8\tau_w}{\rho V^2} \quad \rightarrow$ get $f(Ma) = \frac{f(L^* - L)}{D}$

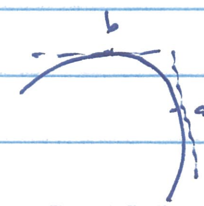
*: (choked) Ma=1 state $\qquad \frac{T}{T^*}$, $\frac{V}{V^*}$, $\frac{p}{p^*}$ $\qquad$ D.2



Rayleigh

$\delta q \neq 0$, $\tau_w = 0 \qquad$ state a $\quad Ma = 1$

state b $\quad Ma = \frac{1}{\sqrt{k}}$



D.3

a: Ma=1 reference state $\qquad \frac{p}{p_a}$, $\frac{T}{T_a}$, $\frac{\rho_a}{\rho}$, $\frac{V_a}{V}$

Normal shock waves

$Ma\downarrow$, $p\nearrow$, $p_0\downarrow$,

x | y $\qquad \frac{p_y}{p_x}$, $\frac{T_y}{T_x} \qquad Ma_y^2 \,(Ma_x^2 \qquad T\nearrow$, $T_0 =$, $\rho\nearrow$, $V\downarrow$

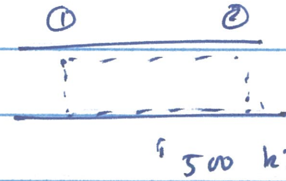
D.4

Ex 11.55

$p_1 = 200\ kPa$

$T_1 = 500K$

$V_1 = 400\ m/s$



Find p_2, T_2, V_2

T-s diag

SI

$$Ma_1 = \frac{400}{\sqrt{(286.9)(500)(1.4)}} = 0.893$$

Fig D.1 → $\frac{T_1}{T_{01}} = 0.86$ → $T_{0,1} = 580K$

Energy eq: $q_{net,in} = h_{0,2} - h_{0,1} = c_p(T_{0,2} - T_{0,1})$

$$\Rightarrow T_{0,2} = -\frac{q_{net,out}}{c_p} + T_{0,1} = -\frac{500,000}{1004} + 580 = 82\ K$$

Fig D.3, $Ma_1 = 0.893$ $\frac{T_{0,1}}{T_{0,a}} = 0.99$, $\frac{p_1}{p_a}, \frac{T_1}{T_a}, \frac{V_1}{V_a}$.

$$\frac{T_{0,2}}{T_{0,a}} = \frac{T_{0,2}}{T_{0,1}} \frac{T_{0,1}}{T_{0,a}} = \frac{82}{580} \times 0.99 = 0.14$$

From D8 read off $Ma_2 = 0.18$ → $\frac{p_2}{p_a}, \frac{T_2}{T_a}, \frac{V_2}{V_a}$.

So $p_2 = \frac{p_2}{p_a} \cdot \frac{p_a}{p_1}\ p_1 = (2.3)\left(\frac{1}{1.14}\right) 200 = 404\ kPa$

$T_2 = \frac{T_2}{T_a} \cdot \frac{T_a}{T_1}\ T_1 = (0.17)\left(\frac{1}{1.02}\right) 500 = 83K$ $< T_{0,2}$ # inaccuracy in graph

$V_2 = \frac{V_2}{V_a} \cdot \frac{V_a}{V_1} \cdot V_1 = (0.07)\left(\frac{1}{0.9}\right) 400 = 31\ m/s$

Can do better using formulas