MAE101B: Advanced Fluid Mechanics Winter Quarter 2020

`http://web.eng.ucsd.edu/~sgls/MAE101_2020/`

Quiz I

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. The continuum description
   - exhibits a linear relation between strain and stress.
   - is valid on sub-atomic scales.
   - means that all equations are one-dimensional.
   - means that variations in properties are smooth enough to use calculus.
   - is a statement about infinite sets.
2. In a fluid at rest,
   - the shear stress points in the normal direction.
   - gage pressure is the same as absolute pressure.
   - the buoyancy force acts horizontally.
   - the force depends linearly on viscosity.
   - pressure is a linear (technically affine) function of depth for homogeneous fluids.
3. In a steady flow,
   - the Bernoulli constant is the same on all streamlines.
   - the acceleration of fluid elements vanishes everywhere.
   - pressure is a linear (technically affine) function of depth for homogeneous fluids.
   - streamlines are the same as particle paths.
   - normal acceleration is inversely proportional to the radius of curvature of a streamline.

2 (7 points) An iceberg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?

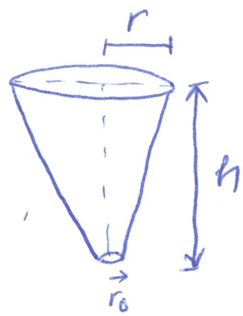
(2) weight : $\rho_I V_I g$ down

(2) Buoyancy : $\rho_o V_s g$ up

$$\% \text{ submerged} = \frac{V_s}{V_I} = \frac{\rho_I g}{\rho_o g} \quad \text{since } W = B \quad (1)$$

$$= \frac{\rho_I}{\rho_o} = \frac{0.917}{1.025} = 89.46\% \quad (2)$$

3 (10 points) A conical container whose sides make an angle of 60° with the vertical is filled with water. An orifice of radius 1.25 mm at the bottom of the cone lets water flow out. If the water depth is initially 80 cm, how long does it take for the water level to go down to 20 cm? State your assumptions clearly. [Use $\rho = 1000$ kg m^{-3}, $g = 9.81$ ms^{-2}.]



$$\tan 60^\circ = \frac{r}{h} \Rightarrow r = \sqrt{3}h$$

From Bernoulli,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (2)$$

Assume :- Steady (1)
- Inviscid (1)

$P_1 = P_2 = 0$ $\quad z_1 = h, \; z_2 = 0$

Also, ~~$A_1 << A_2$~~ $A_2 << A_1 \Rightarrow V_1 << V_2$

$V_1 = -\dot{h}$ $\quad$ From Bernoulli, $V_2 = \sqrt{2gh}$

(2) $A_1 V_1 = A_2 V_2 \Rightarrow \pi r^2 (-\dot{h}) = \pi r_0^2 \sqrt{2gh}$

$$\Rightarrow (3h^2)(-\dot{h}) = r_0^2 \sqrt{2gh}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{\sqrt{2g} \cdot r_0^2}{3} h^{-3/2} \quad (2)$$

$$h^{3/2} dh = -\frac{\sqrt{2g} \cdot r_0^2}{3} dt$$

Integrating from $t=0$ to $t=t$,

$$\int_{h_0}^{h} h^{3/2}\,dh = \frac{-\sqrt{2g}\cdot r_0^2}{3}\int_0^t dt$$

$$\frac{2}{5}h^{5/2}\Big|_{h_0}^{h} = -\frac{\sqrt{2g}\cdot r_0^2}{3}t$$

$$\Rightarrow h^{5/2} - h_0^{5/2} = -\frac{5}{6}\cdot\sqrt{2g}\cdot r_0^2 t$$

$$\Rightarrow t = \frac{(h_0^{5/2} - h^{5/2})\cdot 6}{5\cdot\sqrt{2g}\cdot r_0^2}$$

$h_0 = 80\,cm = 0.8\,m$

$h = 20\,cm = 0.2\,m$

$g = 9.81\,m/s^2$

$r_0 = 1.25\,mm = 1.25\times10^{-3}\,m$

$$\therefore t = \frac{[(0.8)^{5/2} - (0.2)^{5/2}]\cdot 6}{5\cdot\sqrt{2\times9.81}\cdot(1.25\times10^{-3})^2}$$

$= 96{,}149\,s = 26\,hrs\ 42\,mins\ 29\,s$ (2)