

## Quiz II

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. The First Law of thermodynamics in fluids mechanics

- can be viewed as generalizing Bernoulli's equation.
- is Newton's Second Law times velocity.
- ignores gravity.
- requires no shaft work.
- is a vector equation.

2. Circulation

- is zero for viscous flows.
- is a vector.
- is not defined for unsteady flows.
- can be calculated by integrating the pressure.
- is the integral of vorticity over a bounding surface.

3. Being able to scale up from model to full-size requires

- luck.
- the same density fluid.
- dynamic and geometric similarity.
- knowing the torque acting on the model.
- incompressible flow.

2 (7 points) Ships generate waves in their wake and this is one of the mechanisms that induces drag. Derive a nondimensional parameter relating drag  $D$  (a force) for a ship of length  $L$  moving at velocity  $V$  when the acceleration due to gravity is  $g$ . It is easiest to obtain a number proportional to  $L$  (the Froude number). What is its value for a Nimitz class aircraft carrier (length at waterline 317 m, speed 30 knots)? What about for a duck (estimate  $L$  and  $U$ )? [One nautical mile = 1,852 m.]

$$\left. \begin{array}{l} D: MLT^{-2} \\ L: L \\ V: LT^{-1} \\ g: LT^{-2} \end{array} \right\} \begin{array}{l} K=4, r=3 \\ 1 \pi \text{ group} \\ \pi_1 = \frac{Lg}{V^2} \end{array}$$

$$\text{Nimitz: } \frac{317 \times 10}{(30 \times 1.852 \times 10^{-3} / 3600)^2} = 13.3$$

$$\text{Duck: say } \begin{array}{l} L = 20 \text{ cm} \\ v = 0.5 \text{ ms}^{-1} \end{array} \quad \pi_1 = \frac{0.2 \times 10}{(0.5)^2} = 8$$

Note: Couldn't non-dimensionalize  $D$ .

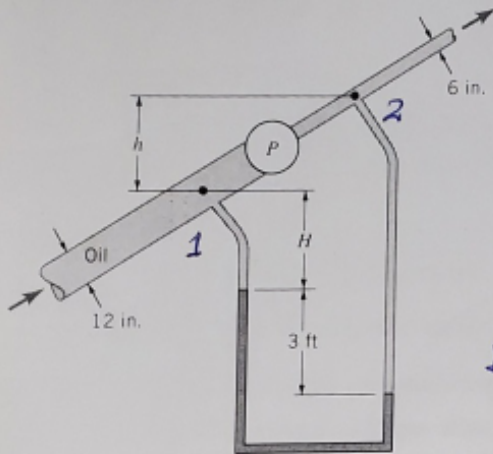
needed another parameter, e.g.  $f: ML^{-3}$

$$\pi_2 = \frac{D}{f g L^3}$$

Froude number: usually defined as  $\frac{V}{\sqrt{g L}}$

$$\left[ \begin{array}{l} \text{Nimitz: } 0.275 \\ \text{Duck: } 0.35 \end{array} \right]$$

3 (10 points) Oil (SG = 0.92) flows in an inclined pipe at a rate of  $4 \text{ ft}^3/\text{s}$  as shown in the figure below. If the differential reading in the mercury manometer is 3 ft, calculate the power that the pump supplies to the oil if head losses are negligible.



$$\frac{P_{out}}{\gamma} + \frac{V_{out}^2}{2g} + Z_{out}$$

$$= \frac{P_{in}}{\gamma} + \frac{V_{in}^2}{2g} + Z_{in} + h_s - h_L$$

In this case,

$$\frac{P_2}{\gamma_{oil}} + \frac{V_2^2}{2g} + Z_2 = \frac{P_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + Z_1 + h_s - h_L$$

$$Q = 4 \text{ ft}^3/\text{s} \quad \therefore V_1 = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{4}{\frac{\pi (12)^2}{4}} = 5.09 \text{ ft/s}$$

$$V_2 = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{4}{\frac{\pi (6)^2}{4}} = 20.30 \text{ ft/s}$$

$$\text{Now, } P_1 + \gamma_{oil} \cdot H + \gamma_{Hg} \cdot 3 - \gamma_{oil} (3 + H + h) = P_2$$

$$\therefore \frac{P_1}{\gamma_{oil}} + 3 \frac{\gamma_{Hg}}{\gamma_{oil}} - 3 - h = \frac{P_2}{\gamma_{oil}}$$

Bernoulli becomes,

$$\frac{P_1}{\gamma_{oil}} + 3 \frac{\gamma_{Hg}}{\gamma_{oil}} - 3 - h + \frac{V_2^2}{2g} + Z_2 = \frac{P_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + Z_1 + h_s$$

$$\Rightarrow h_s = 50.38 \text{ ft}$$

$$\begin{aligned} \dot{W}_{shaft} &= \gamma_{oil} Q h_s = SG_{oil} \gamma_{H_2O} \cdot Q h_s \\ &= 11581.5 \frac{\text{ft}}{\text{s}} = 21 \text{ hp} \end{aligned}$$