MAE101B: Advanced Fluid Mechanics Winter Quarter 2020

http://web.eng.ucsd.edu/~sgls/MAE101_2020/

Quiz IV

This is a 50 minute closed-book exam. Please put your name on the top sheet. Answer all three questions. Explain your working and state any assumptions you have made.

1 (3 points) Circle the correct answer.

1. For a perfect gas, entropy
   - remains a constant.
   - is a function of pressure, temperature and electric field.
   - (•) is a thermodynamic function of state.
   - depends only on the area of a nozzle.
   - decreases through shocks.
2. The Mach number
   - (•) depends on the state via temperature alone for an ideal gas.
   - is constant in space for a given flow.
   - is all that is needed to compute the state of the fluid.
   - depends only on the area of a nozzle.
   - is never 1 in a converging-diverging nozzle.
3. Isentropic flow in a duct
   - only has shocks at locations with maximum area.
   - (•) corresponds to vertical lines on a T-s diagram.
   - has constant pressure.
   - has increasing velocity as the duct area decreases in all cases.
   - is always supersonic.

2 (7 points) The stagnation pressure in a Mach 2 wind tunnel operating with air is 900 kPa. A sphere of diameter 1 cm positioned in the wind tunnel has a drag coefficient of 0.95. Calculate the drag force on the sphere.

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A_p} \Rightarrow D = C_D\left(\frac{1}{2}\rho V^2\right)\left(\frac{\pi D^2}{4}\right)$$

$$V^2 = Ma^2 c^2 = Ma^2 kRT = Ma^2 k\frac{p}{\rho}$$

$$\therefore \rho V^2 = Ma^2 kp$$

$$D = C_D\left(\frac{1}{2}Ma^2 kp\right)\frac{\pi D^2}{4}$$

$$= \frac{\pi}{8} C_D k \left(\frac{p}{p_0} \times p_0\right) Ma^2 D^2$$

At $Ma = 2$, $\frac{p}{p_0} = 0.1278$

$$\therefore D = \frac{\pi}{8}(0.95)(1.40)(0.1278)(900\times10^3)\,2^2 \times (0.01)^2$$

$$\Rightarrow \boxed{D = 24\,N}$$

3 (10 points) A converging-diverging nozzle discharges air at supersonic speeds. The flow has velocity $V_1 = 150$ ms^{-1}, pressure $p_1 = 100$ kPa and temperature $T_1 = 20°C$ at section 1 upstream of the throat. The area at the throat is 0.1 m^2. Compute the mass flow rate.

$$c_1 = \sqrt{KRT_1} = \sqrt{1.4 \times 287 \times 293} = 343.11 \text{ m/s}$$

$$\therefore Ma_1 = \frac{V_1}{c_1} = \frac{150}{343.11} = 0.44$$

From chart, $\frac{P_1}{P_{01}} = 0.88$ $\quad$ $\frac{T_1}{T_{01}} = 0.97$

$$\Rightarrow P_{01} = 113.64 \text{ kPa} \qquad T_{01} = 302.06 \text{ K}$$

$P_{02} = P_{01}$ and $T_{02} = T_{01}$

At throat $Ma = 1 \Rightarrow A^* = 0.1 \text{ m}^2$

$$\therefore \dot{m} = \frac{P_{02} A^*}{\sqrt{R T_{02}}} \sqrt{K} \left(\frac{2}{K+1} \right)^{\frac{K+1}{2(K-1)}}$$

$$= \frac{113.64 \times 10^3 \times 0.1}{\sqrt{287 \times 302.06}} \sqrt{1.4} \left(\frac{2}{2.4} \right)^{\frac{2.4}{2(0.4)}}$$

$$= 26.43 \text{ kg/s}$$

Units and constants

1 hp = 550 lb ft/s
1 in = 2.54 cm
1 mile = 1609 m
1 mph = 88 ft/min
Acceleration of gravity: $g = 9.81$, $g = 32.2$ ft/s^2
°R = °F + 459.67

Material properties (SI unless otherwise stated)

Air: $\rho = 1.23$ and $\mu = 1.79 \times 10^{-5}$ at 15°C
Air: $\rho = 2.38 \times 10^{-3}$ slugs/ft^3 and $\mu = 3.47 \times 10^{-7}$ lb·s/ft^3 at 59°F
Air (ideal gas): $c_p = 1004$, $R = 287.1$ (SI), $R = 1{,}716.5$ ft lb/slug °R (BG), $R = 53.35$ ft lb/lbm °R (EE)

Isentropic flow of an ideal gas with $k = 1.4$

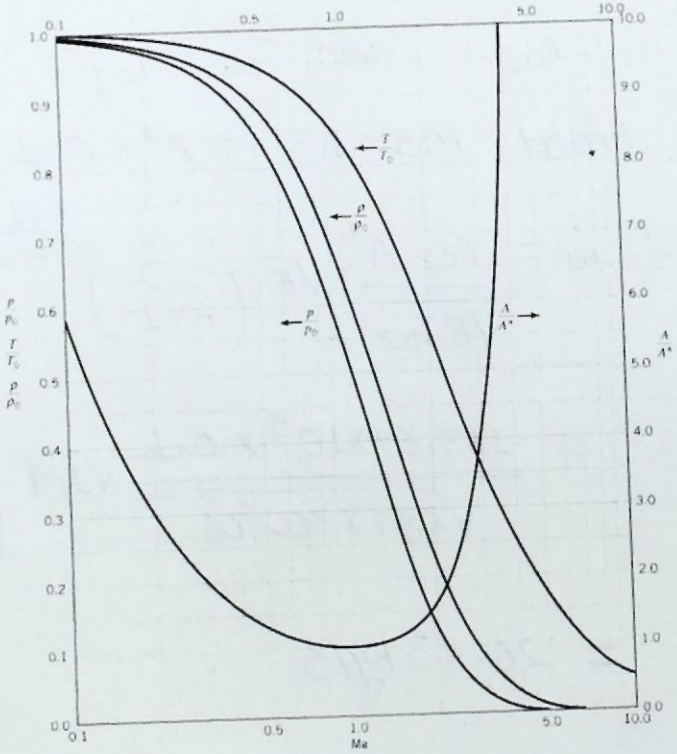

Figure D.1
Graph provided by Dr. Bruce A. Reichert