Review of Basic Mathematics for MAE105

1.Differential and Integral Calculus (Math 20A, 20B)

The following differentiation formulas are helpful:

$f(t)$	$\dot{f}(t)=\frac{d}{dt}f(t)$
$t^n \quad n\neq 0$	nt^{n-1}
$t^0=1$	0
$\sin t$	$\cos t$
$\cos t$	$-\sin t$
$e^t=\exp t$	e^t

Let $f(t)$ and $h(t)$ be differentiable functions of time and c be a constant.

$\frac{d}{dt}(cf)=c\frac{df}{dt}$
$\frac{d}{dt}(f\pm h)=\frac{df}{dt}\pm\frac{dh}{dt}$
$\frac{d}{dt}(fh)=\frac{df}{dt}h+f\frac{dh}{dt}$
$\frac{d}{dt}\left(\frac{f}{h}\right)=\frac{\frac{df}{dt}h-f\frac{dh}{dt}}{h^2}$ if $h(t)\neq 0$
chainrule $\frac{d}{dt}h(f(t))=\frac{dh(q)}{dq}\frac{df(t)}{dt}$ where $q=f(t)$

The following integration formulas are shown for reference. In the table, C denotes the integration constant.

$\int_0^t \tau^n\,d\tau=\frac{t^{n+1}}{n+1}+C$ if $n\neq -1$
$\int_0^t e^\tau\,d\tau=e^t+C$
$\int_0^t \sin\tau\,d\tau=-\cos t+C$

$$\int_0^t \cos\tau \, d\tau = \sin t + C$$

Exercise 1. Perform the following differentiations using the chain rule of differentiation:

(a) $\frac{d}{dt}\left(e^{pt}\right)$,where p is a constant, (d) $\frac{\partial}{\partial x}(\frac{1}{2}k(x-y)^2)$, where k is a constant;

(b) $\frac{\partial}{\partial y}(\frac{1}{2}k(x-y)^2)$, where k is a constant, (e) $\frac{d}{dt}\left(e^{i\omega t}\right)$, where ω is a constant;

(c) $\frac{d}{dt}\left(\sin(\omega t-\phi)\right)$, where ϕ is a constant; (f) $\frac{\partial}{\partial x}\left(\frac{1}{2}k_1(x-y)^2+\frac{1}{2}k_2(z-x)^2\right)$

If you have difficulties in solving the exercise problems, Google "Differential and Integral Calculus" to find inspiring web pages and You Tube lectures.

2.Integration by Parts (Math 20D)

The product rule states that if $f(t)$ and $g(t)$ are differentiable functions, then

$$\frac{d}{dt}[f(t)\,g(t)] = \dot{f}(t)\,g(t) + f(t)\,\dot{g}(t) \quad \text{where } \dot{f} = \frac{df}{dt} \text{ and } \dot{g} = \frac{dg}{dt}.$$

In the notation for indefinite integrals this equation becomes

$$\int \dot{f}(t)\,g(t)\,dt + \int f(t)\,\dot{g}(t)\,dt = f(t)\,g(t).$$

After rearrangement, we obtain the formula *for integration by parts*:

$$\int f(t)\,\dot{g}(t)\,dt = f(t)\,g(t) - \int \dot{f}(t)\,g(t)\,dt.$$

For example, $\int t\sin t\,dt$ we note that $f(t)=t$ and $\dot{g}(t)=\sin t$ where $\dot{f}(t)=1$ and $g(t)=-\cos t$.

$$\int t\sin t\,dt = t(-\cos t) - \int(-\cos t)\,dt = -t\cos t + \sin t + C.$$

3.Linear Ordinary Differential Equations (ODEs) with Constant Coefficients (Math 20D)

First Order Equation: $J\,\dot{q} + a\,q = 0$

Second Order Equation: $m\ddot{h} + k^2\,h = 0$

If you wish to review the solution of linear ODEs with constant coefficients, google will guide you to many lecture notes.